Small-World Connectivity Exhibited in Memristive Nanowires

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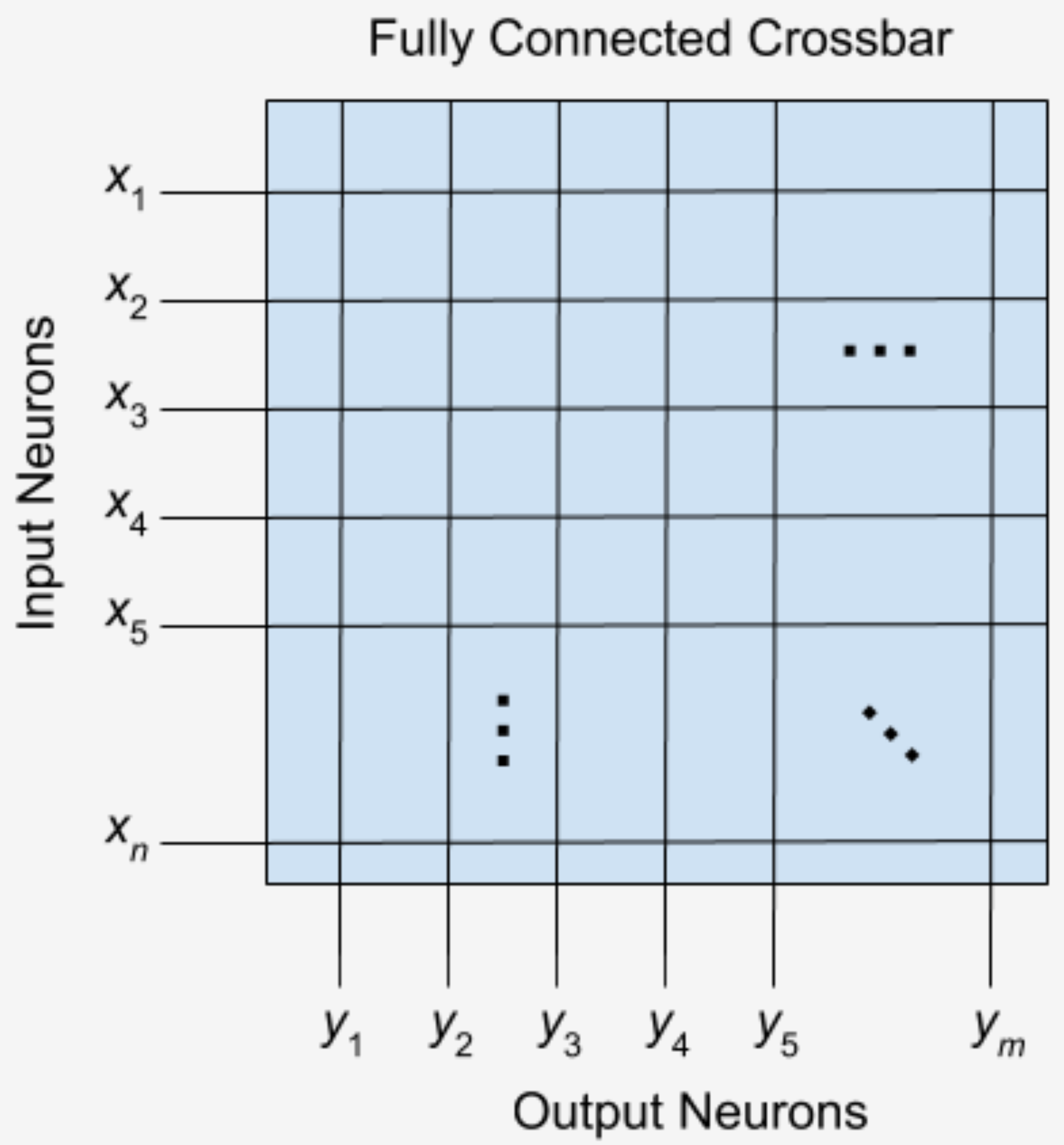
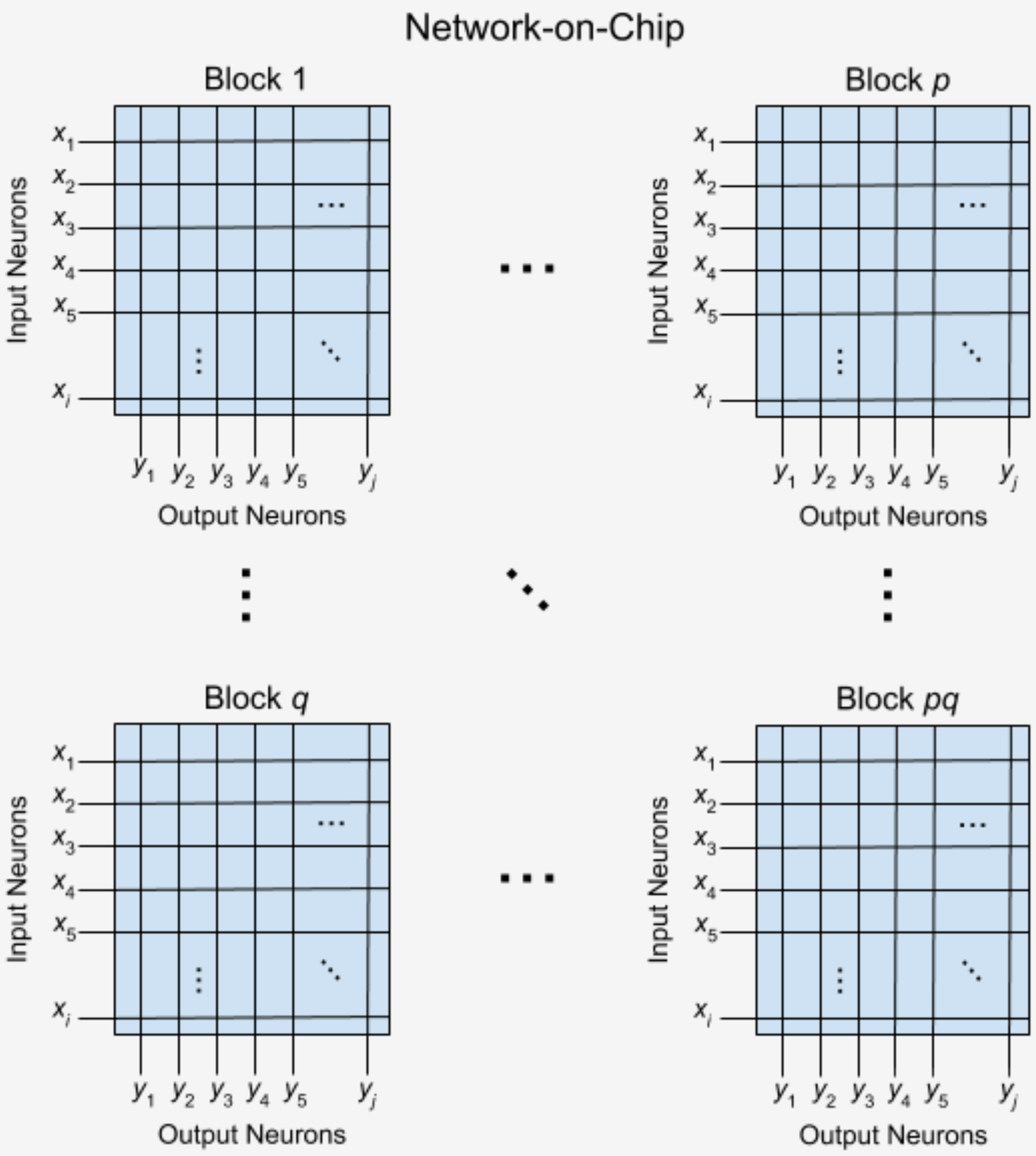


Scaling Requires Sparsity

- **Fully Connected Crossbar**
  - Not sparse
  - Chip size scales quadratically with additional neurons
  - Neuron density decreases as neurons are added
  - Quickly becomes untenable for large numbers of neurons



Not All Sparsity Is Useful

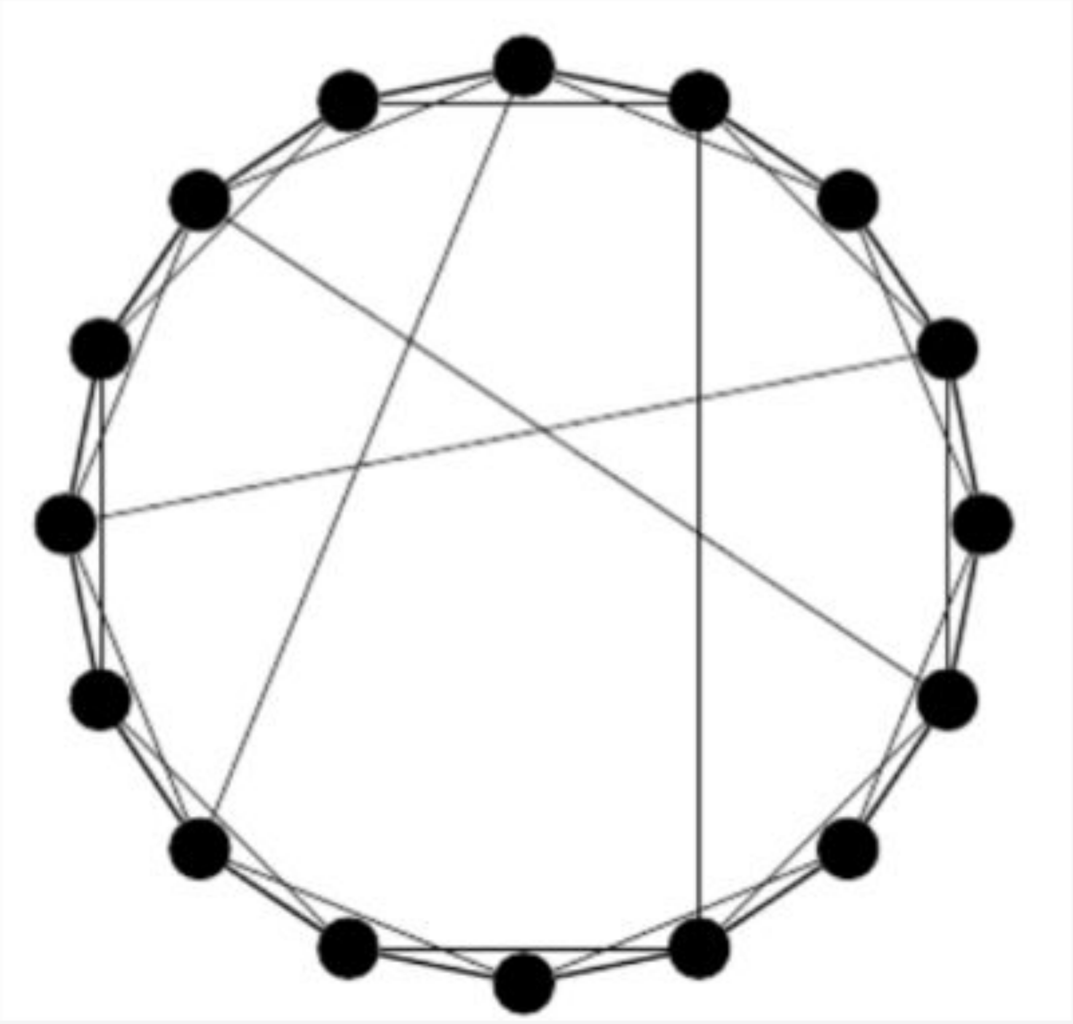
- **Network-on-Chip**
  - Locally **parallel** connectivity
  - Globally **serial** connectivity
  - Network traffic saturates as crossbars are added (deadlock)

The Brain Has Solved This Problem

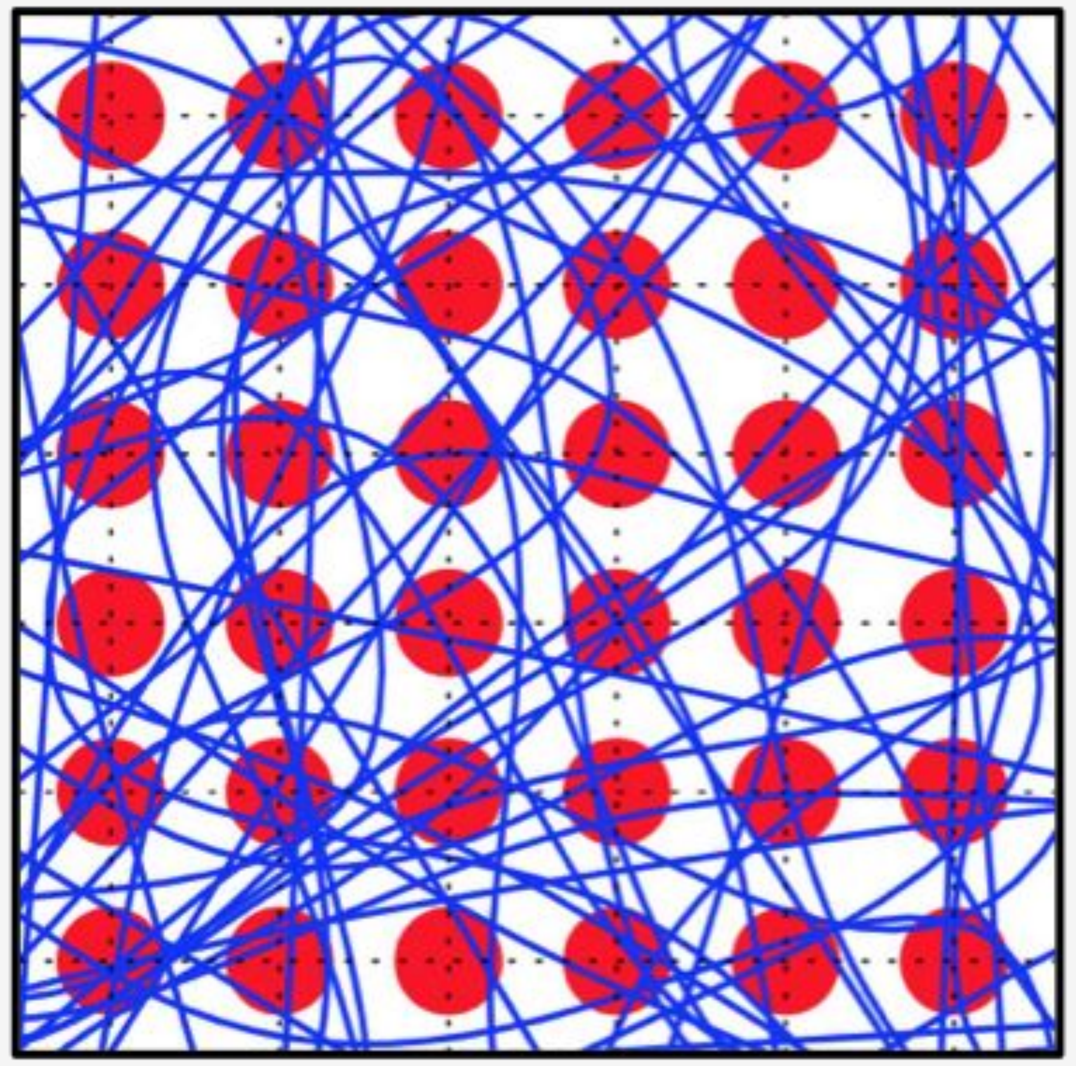
The brain contains roughly 10^{11} neurons and only about 10^{14} synapses [1]. If it were fully connected, there would be an unfeasible 10^{22} synapses. Further, it is highly parallelizable and does not suffer from communication stalls.

Small-World Networks

- Many local connections but few long-range connections
- Balance wiring cost and global efficiency [2]
- Prevalent in the brain, social networks, electric power grids, and connected protein networks, to name a few [3]



Conventional small-world network



The MN3, which we show is small-world

Small-World Networks

- Formally, the small-world coefficient σ is defined by,

$$\sigma = (C / C_r) / (L / L_r),$$

where C and C_r are the square clustering coefficients of the MN^3 and a random bipartite graph with the same number of total connections and arrangements of vertices, respectively, and L and L_r are the average shortest paths between two nodes on the MN^3 and the same random graph. [4]

- If $\sigma > 1$, then the network is small-world.

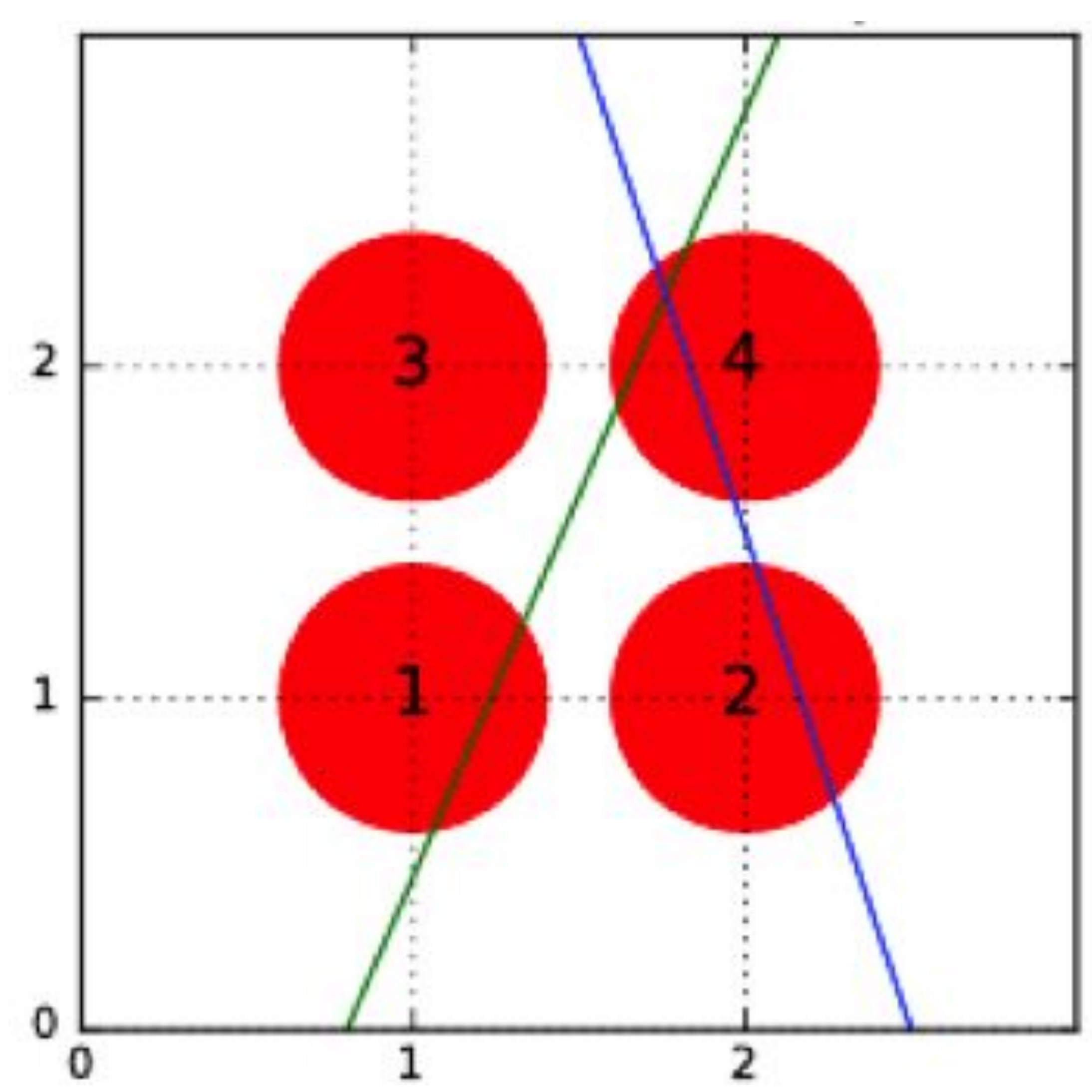
- Typically,

$$L \sim \log(N),$$

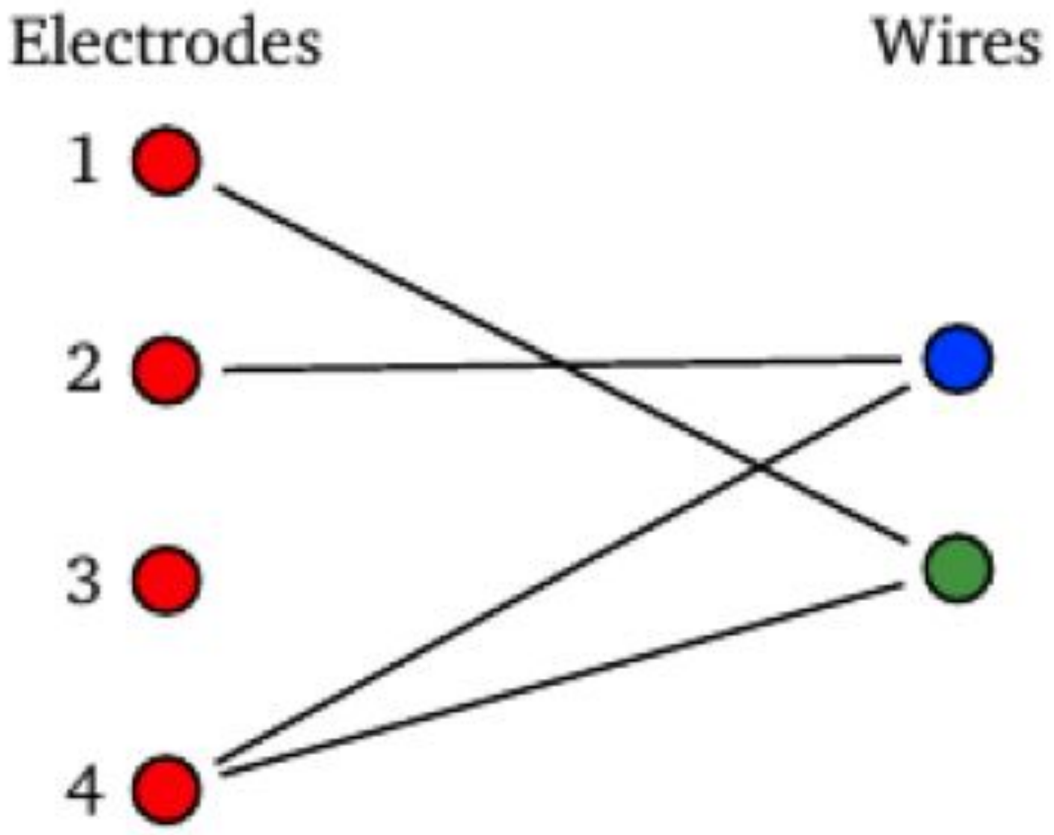
where N is the total number of nodes.

Bipartite Connectivity

- MN^3 connectivity can be represented as a bipartite graph [5]
- One class of vertices comprises the electrodes and the other comprises the wires
- If a wire goes over an electrode, then an edge is drawn between the pair
- Various graph metrics can be calculated

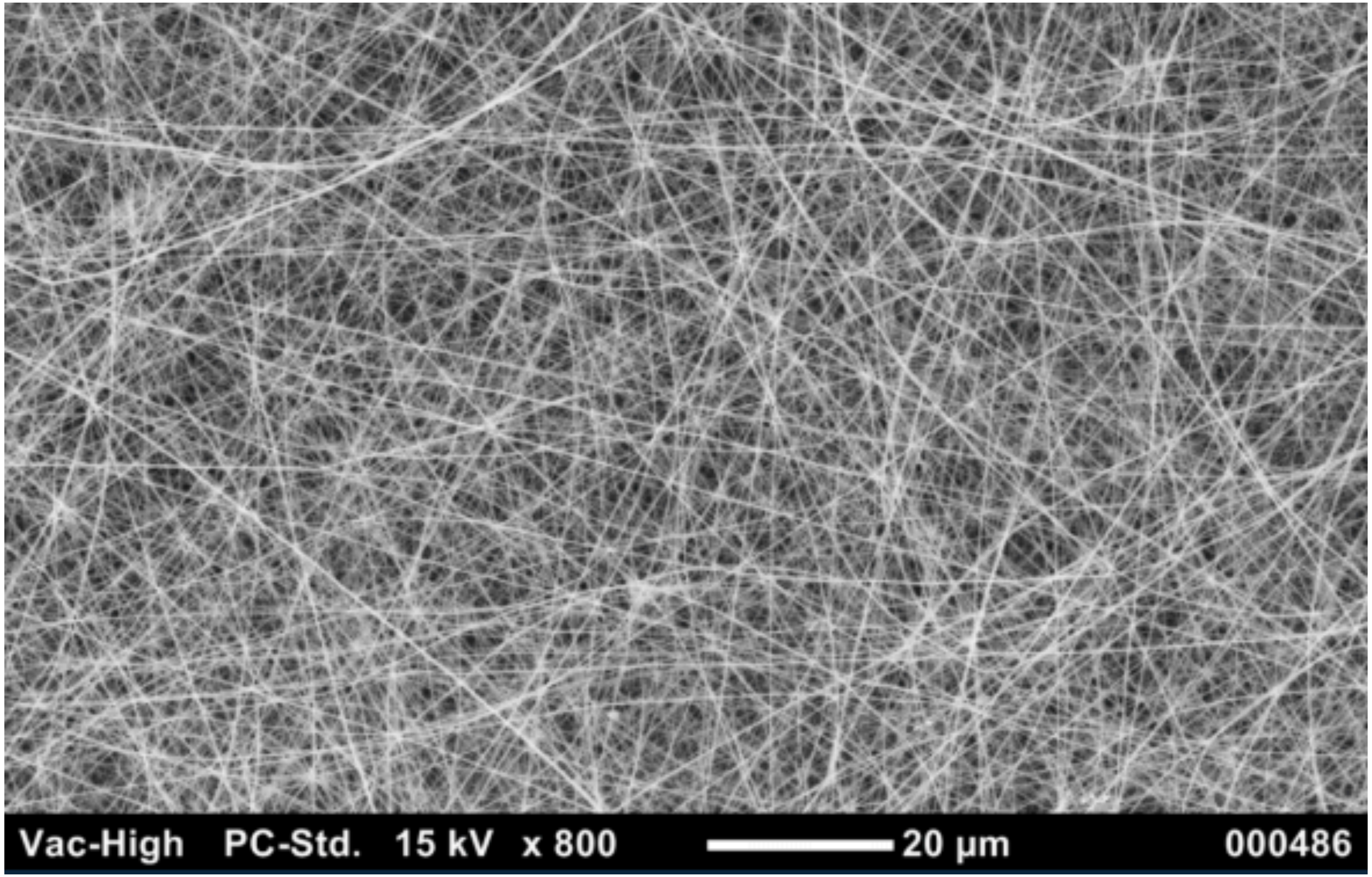


Simple grid with four electrodes and two wires for demonstration purposes



Corresponding bipartite representation

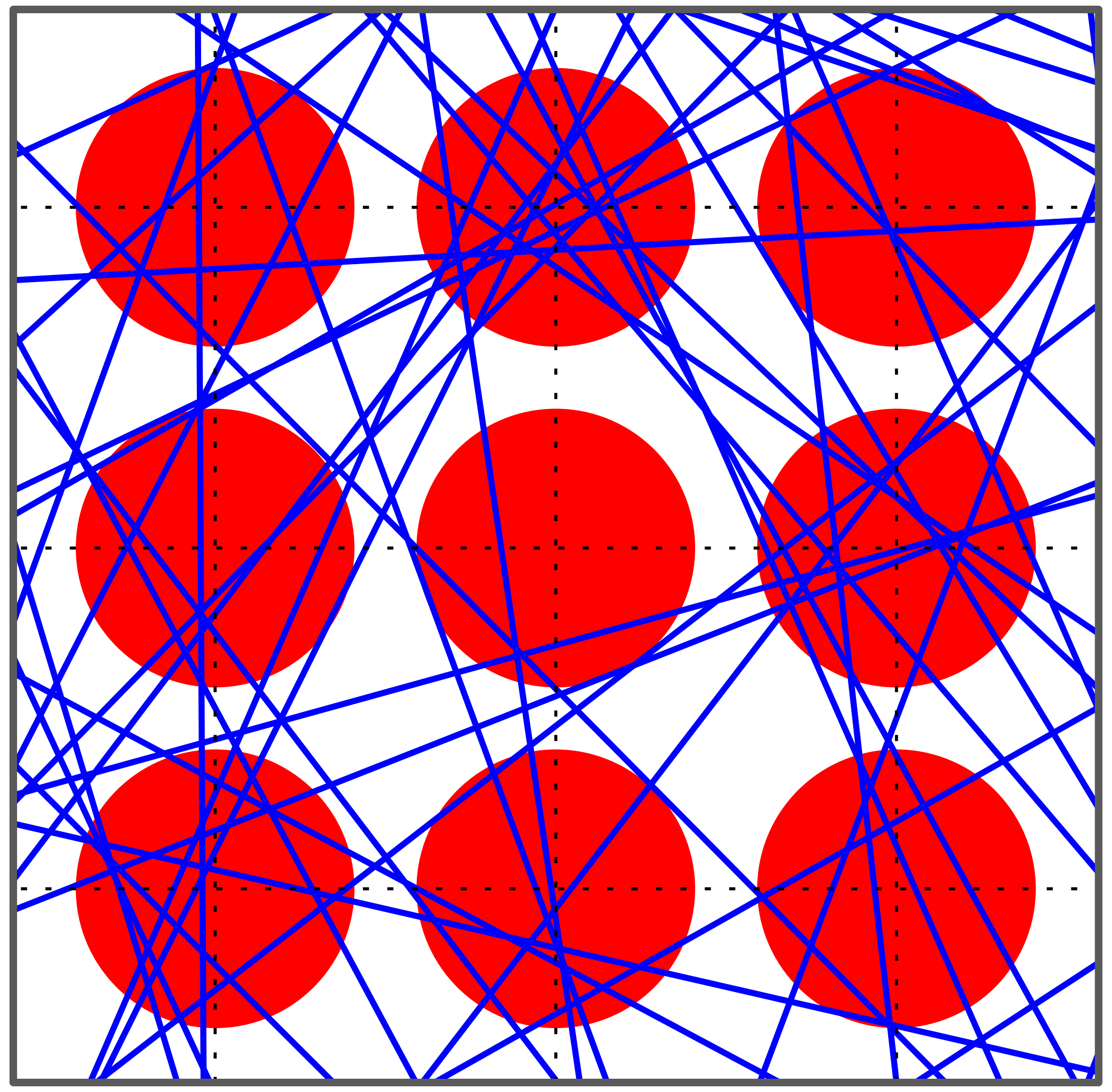
Physical Nanowires



J.C. Nino and J.D. Kendall - PCT/US2015/034414, (2015).

MN³ Nanowire Models

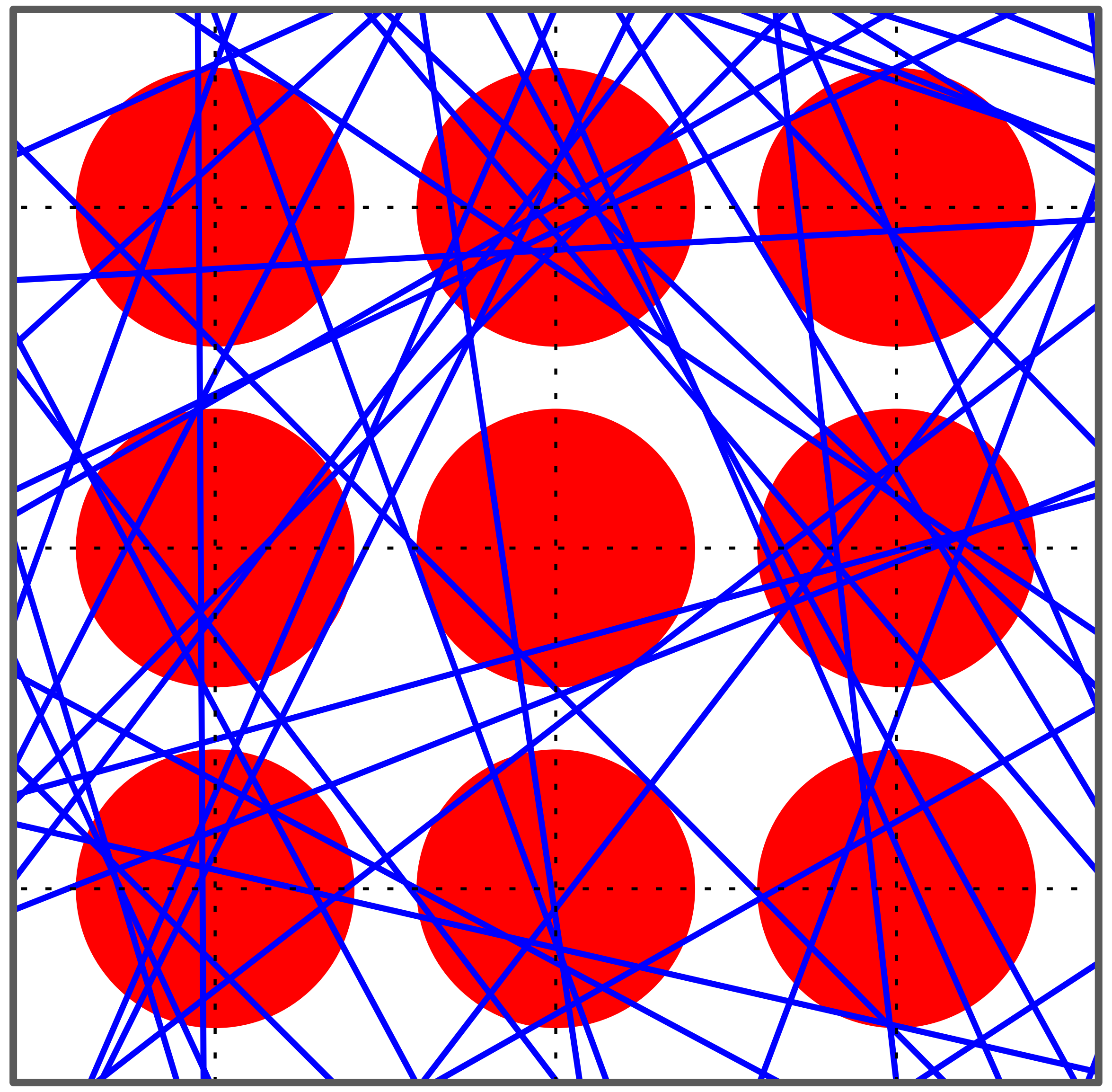
Straight Wire



- Assumes wires are straight
- Wires go from one of the four sides to another unique side at random
- Computationally simple

MN³ Nanowire Models

Straight Wire



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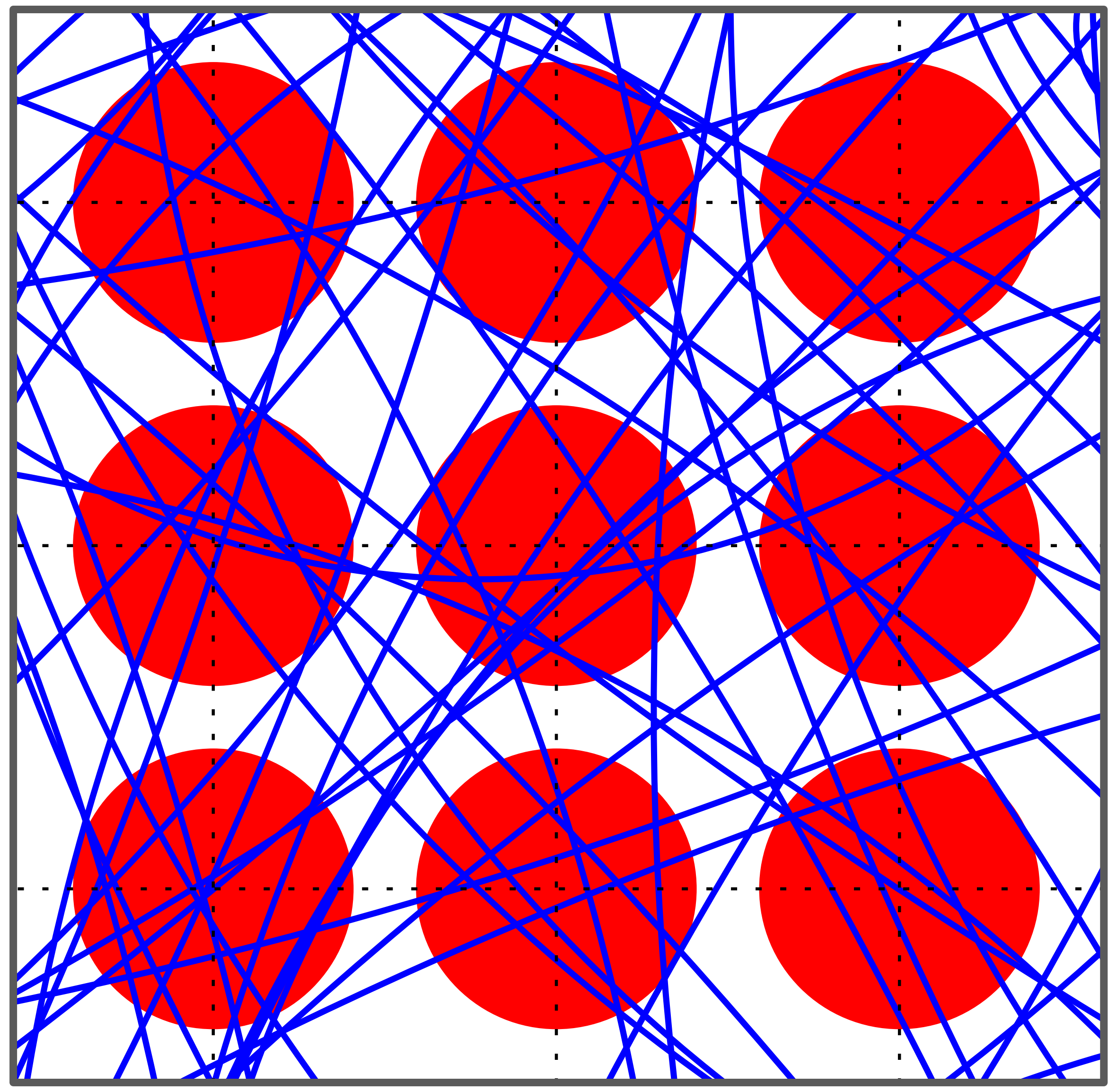
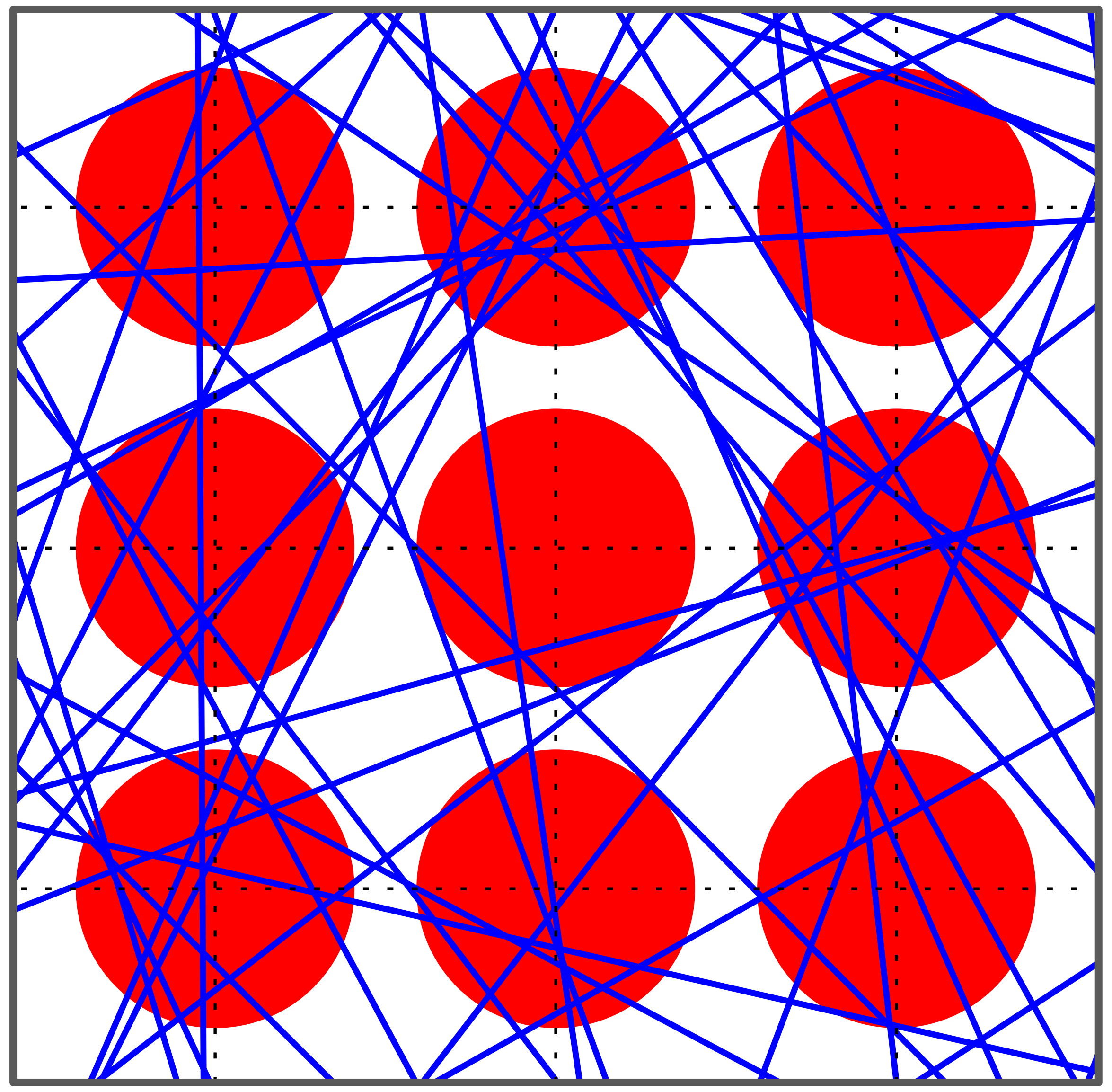
Arc Wire



- Assumes wires are arcs of random radii
- Wires go from one of the four sides to a not necessarily unique side at random

MN^3 Nanowire Models

Straight Wire



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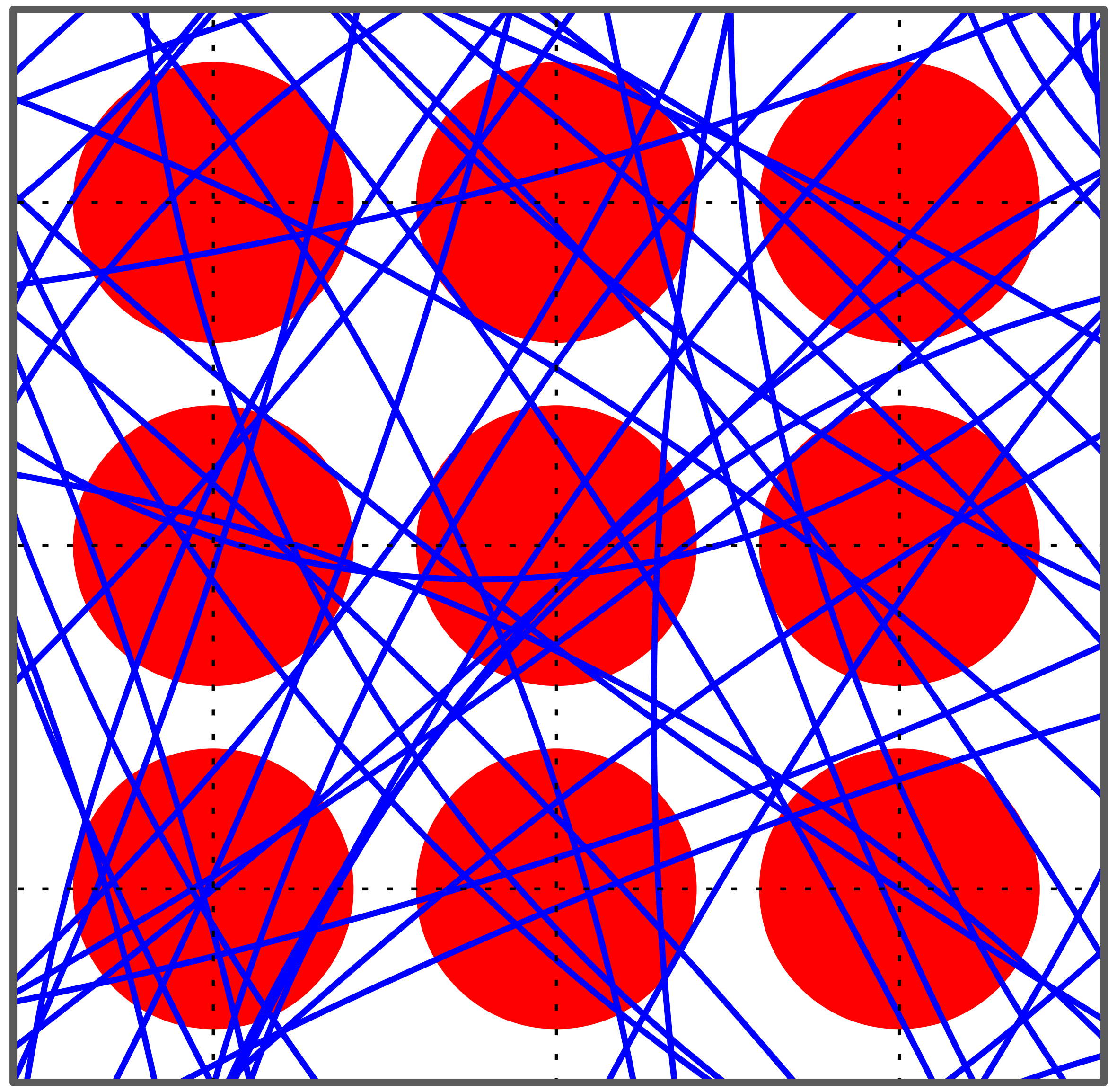

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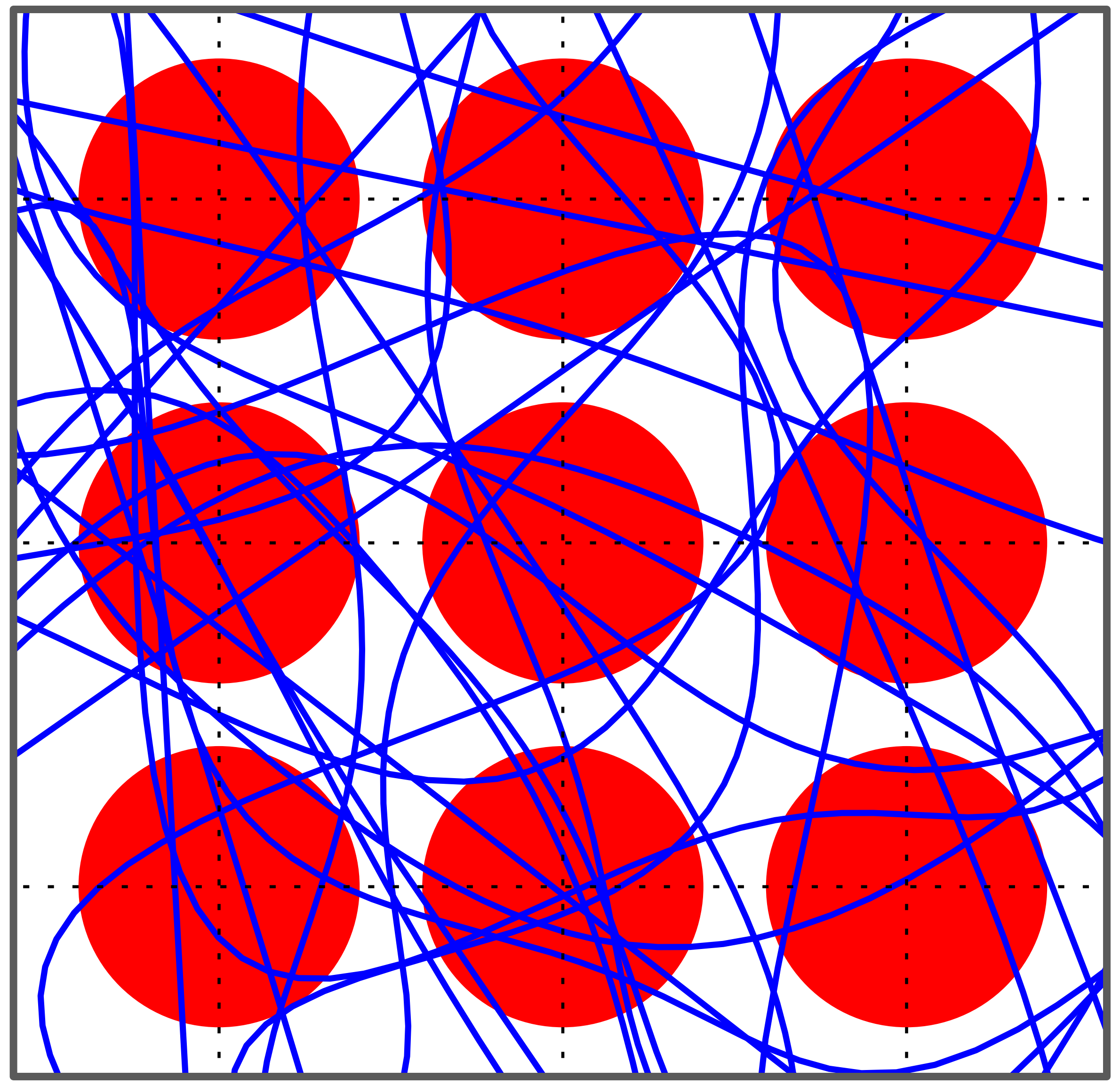
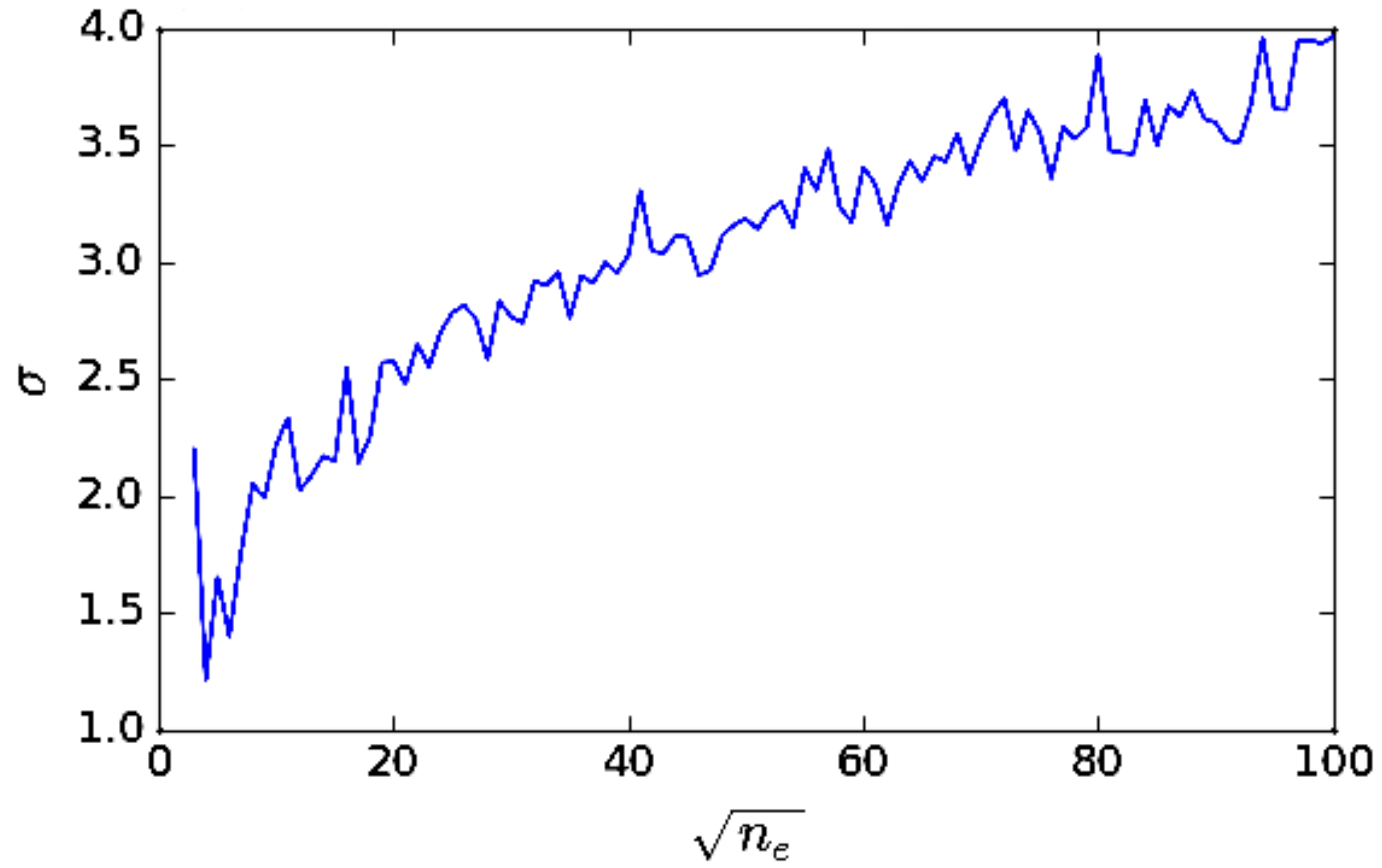
Pink Noise Wire



- Assumes wires follow a path generated by approximate pink noise
- Distances are autocorrelated
- Computationally expensive

Small-Worldness

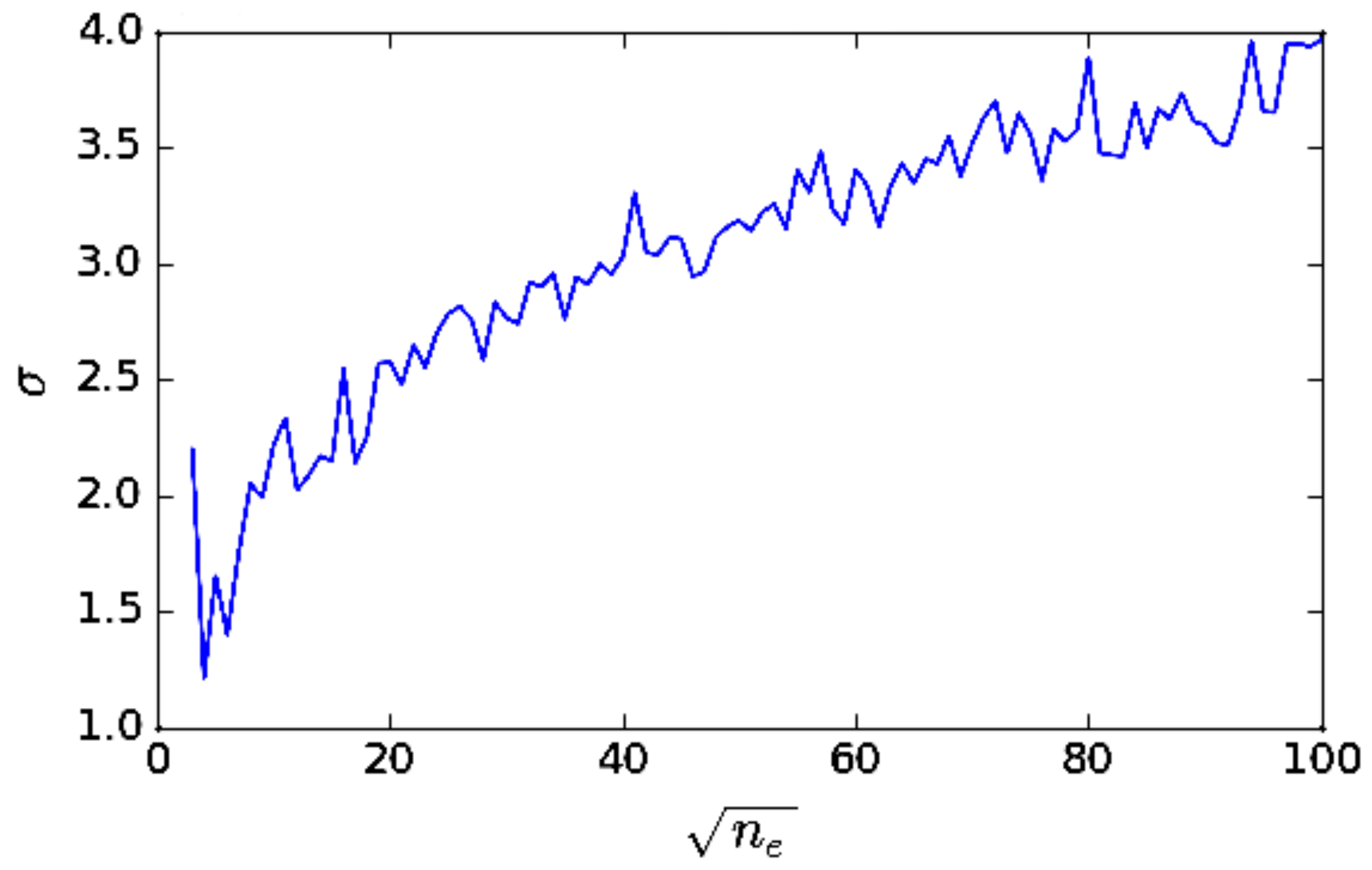
Straight Wire



- σ > 0 for all grid sizes, implying small-worldness
- Increases logarithmically

Small-Worldness

Straight Wire



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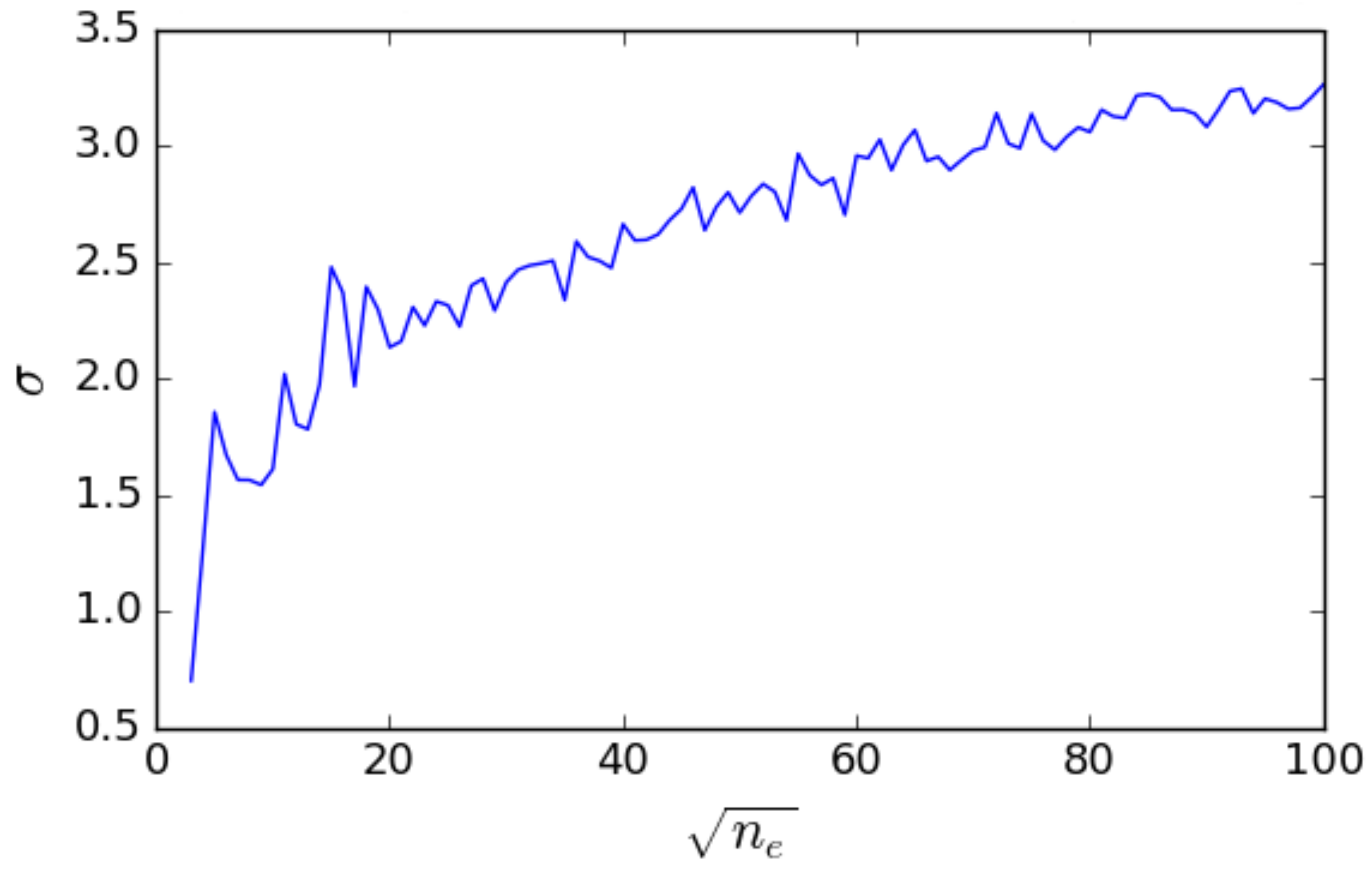
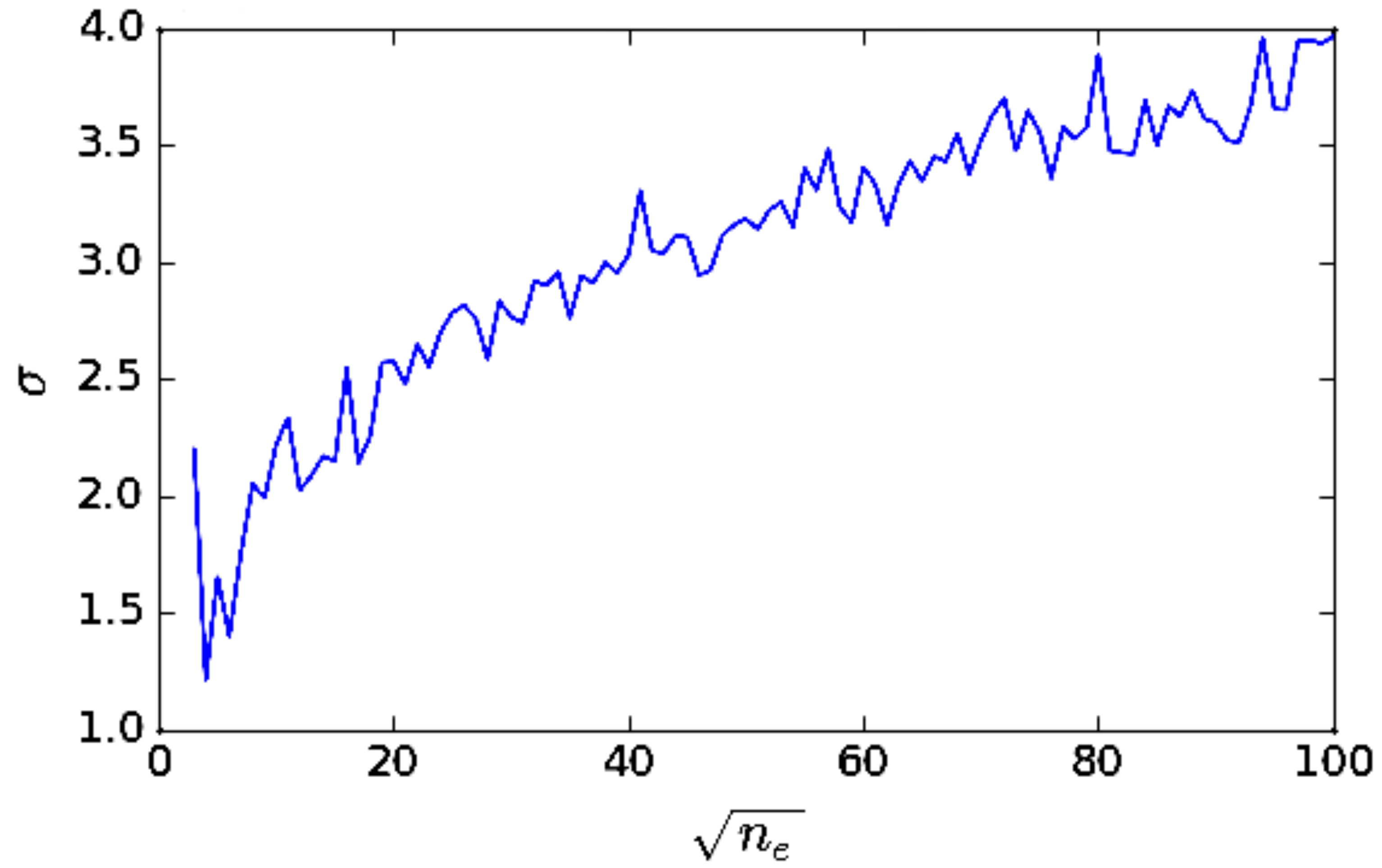
Arc Wire



- Behaves similarly to the straight wire model

Small-Worldness

Straight Wire



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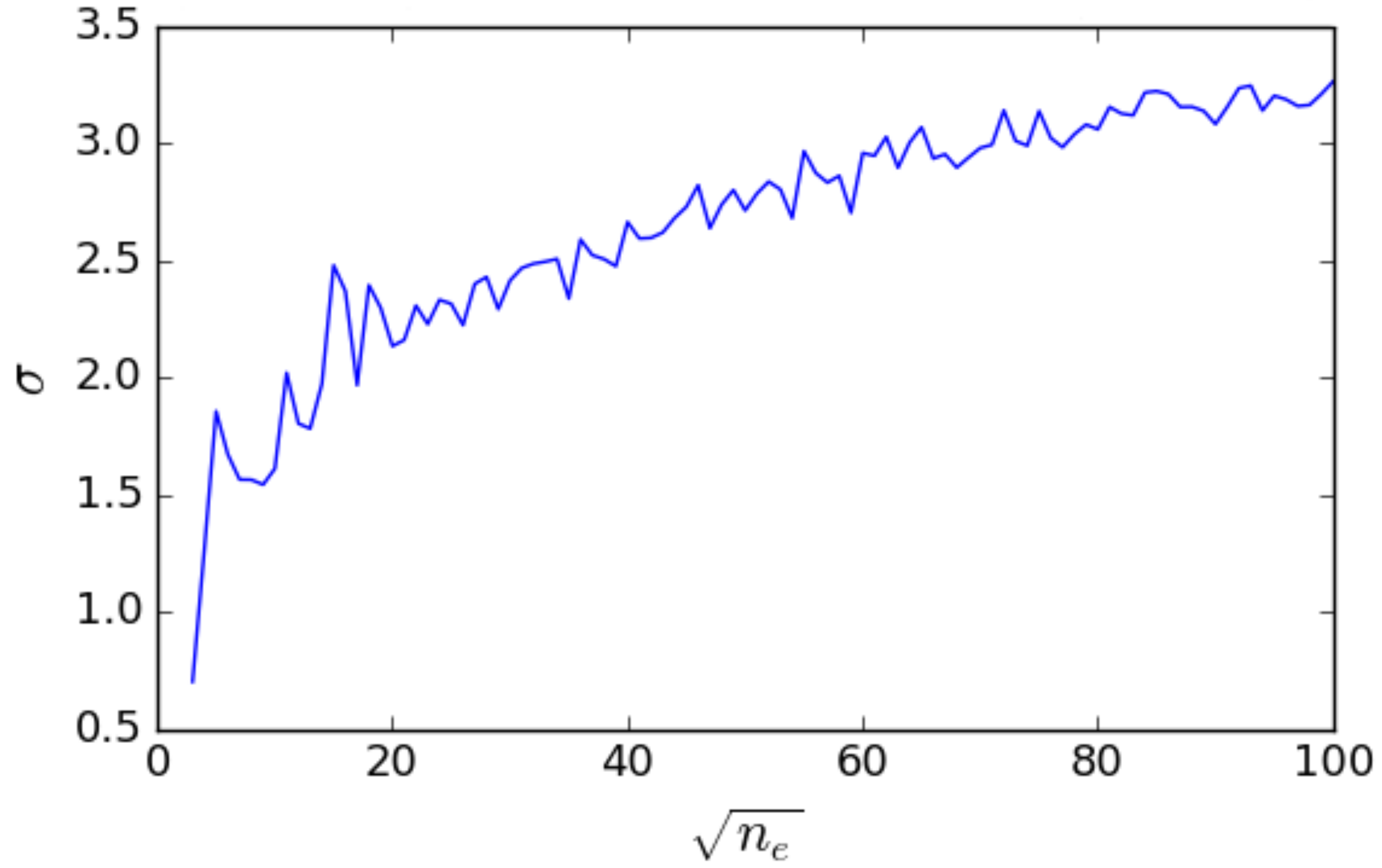
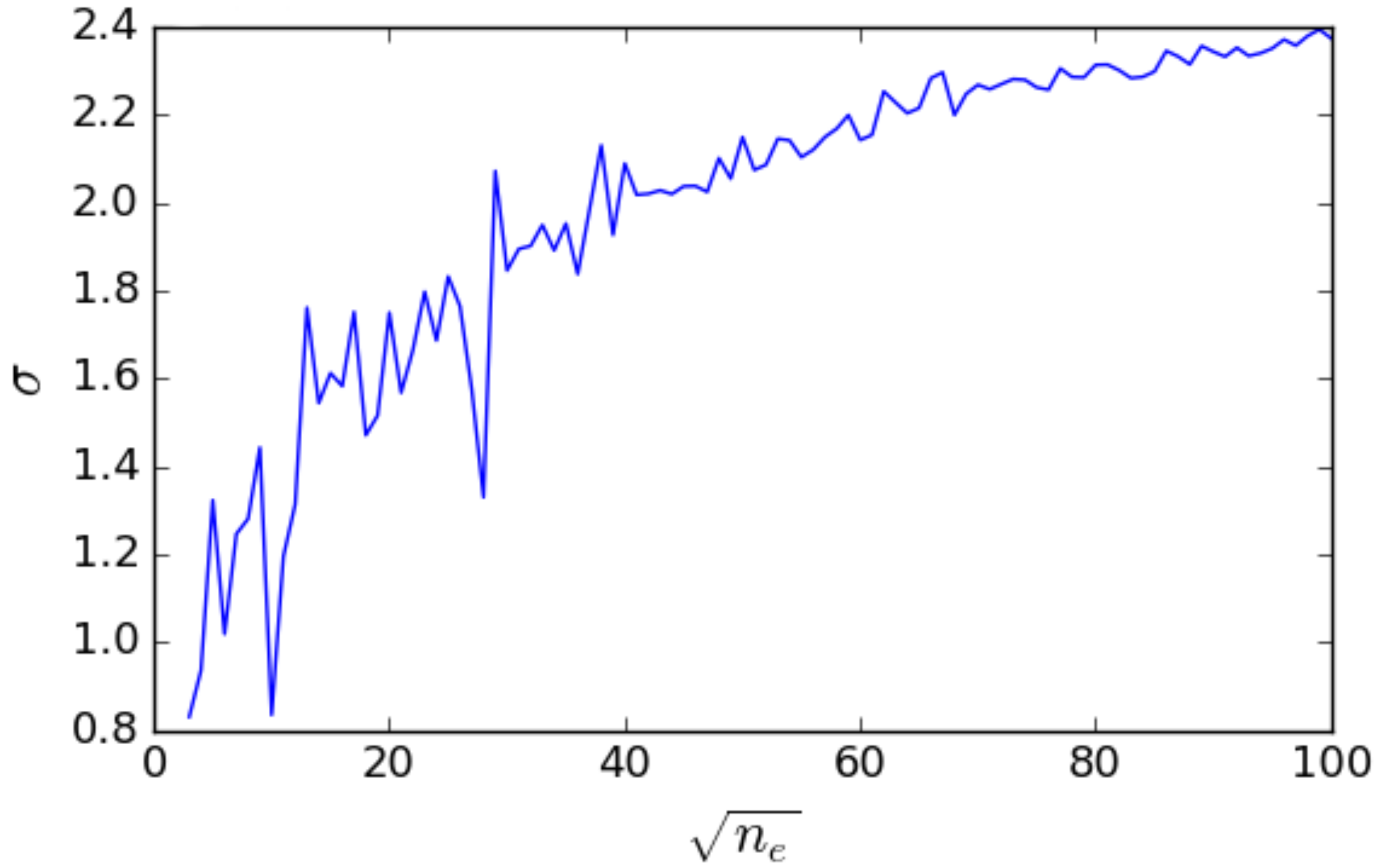
Arc Wire



- Behaves similarly to the straight wire model

Pink Noise Wire



- Similar to other two models but with more noise, which is expected with an increase in degrees of freedom

Future Work: Scale-Free Networks

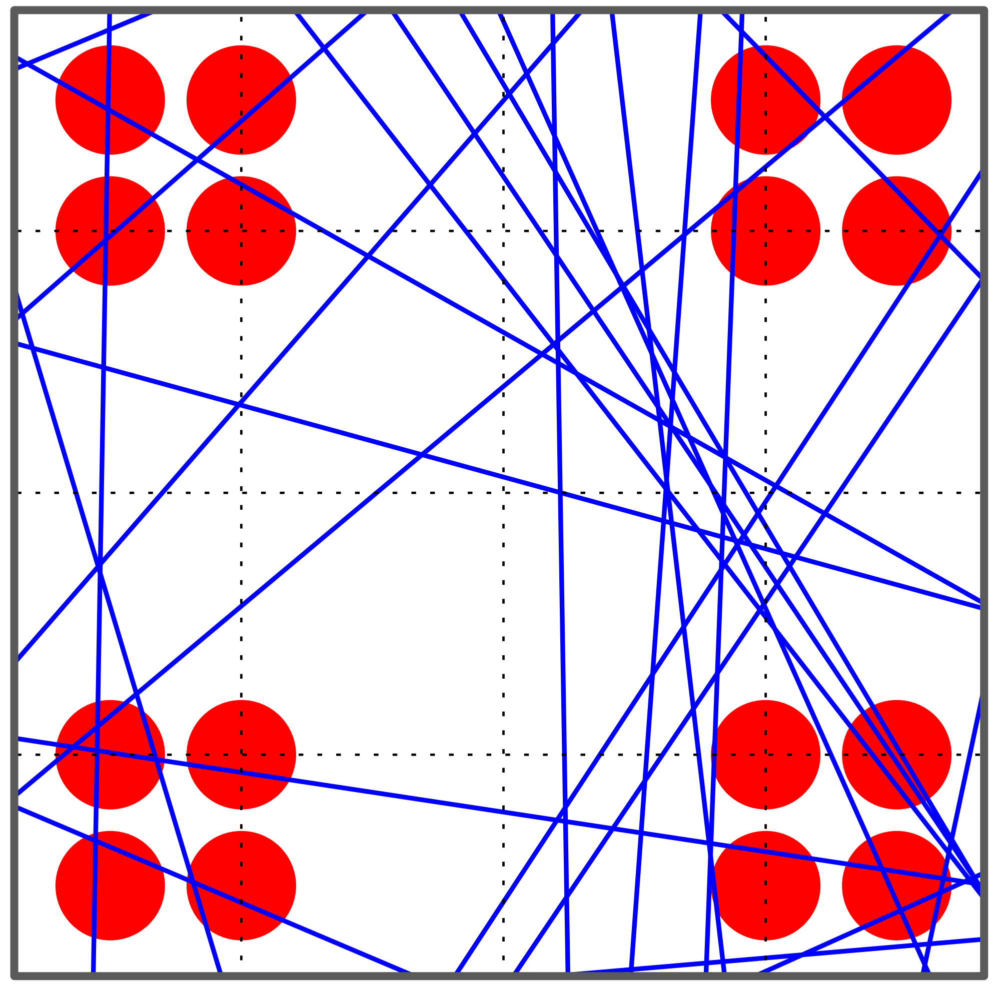
- A scale-free networks, also called ultra small-world networks, are a subset of small-world networks
- The brain is a scale-free network [6].
- Typically,

$$L \sim \log(\log(N)),$$

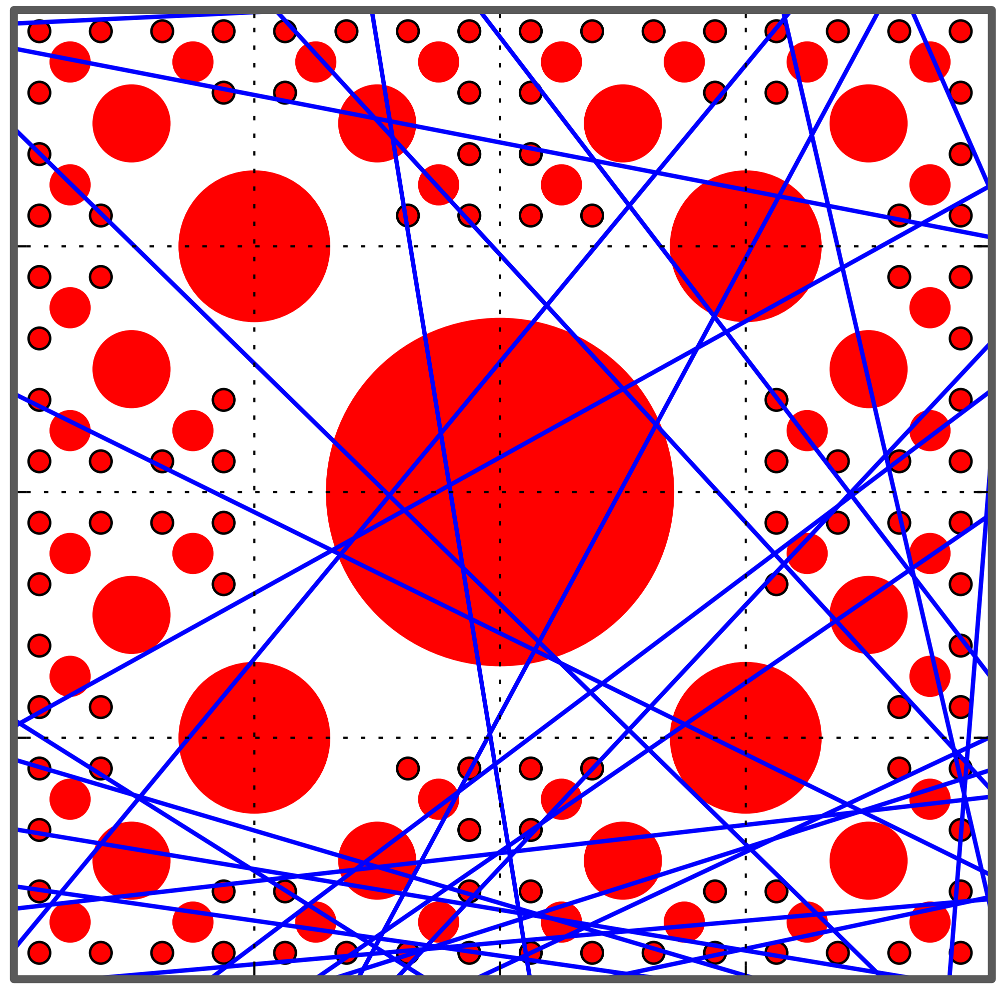
where N is the total number of nodes [7].

Image: https://farm6.staticflickr.com/5572/14840495571_81ca4e1ebe_o.png

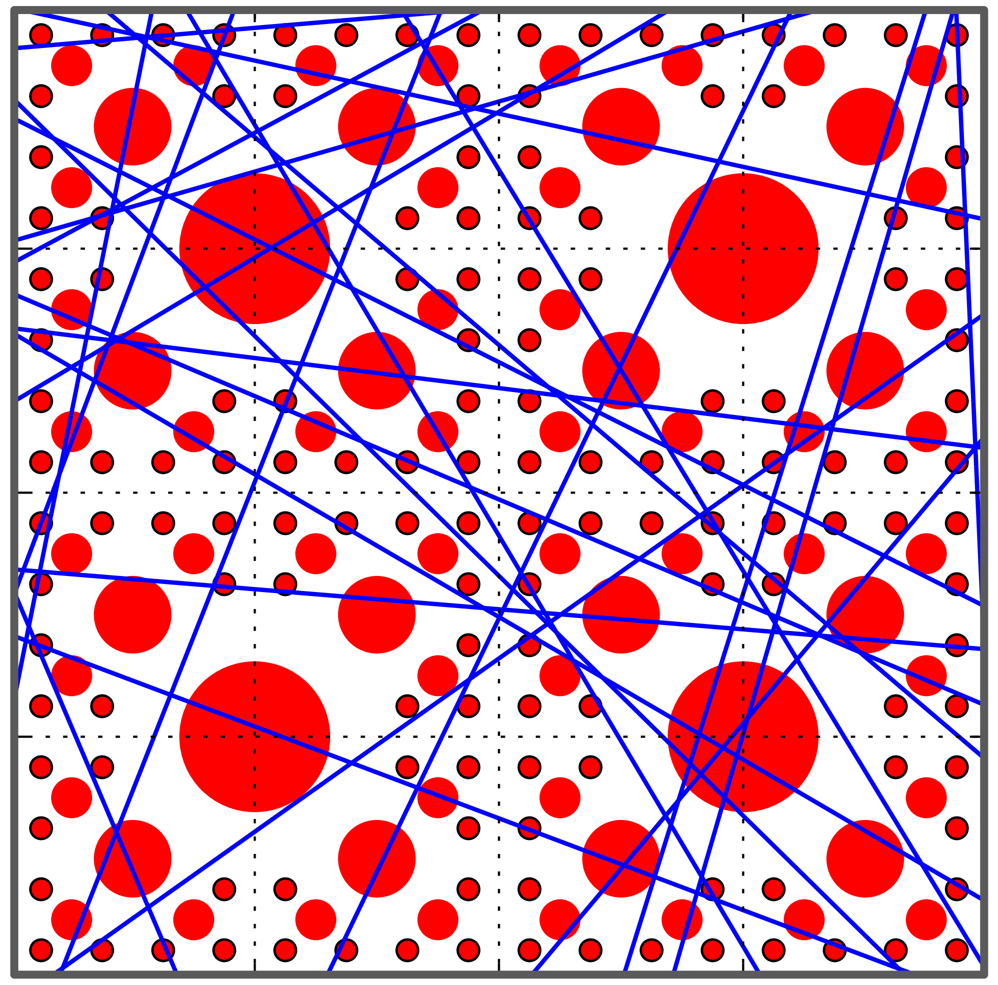
Future Work: Scale-Free Networks



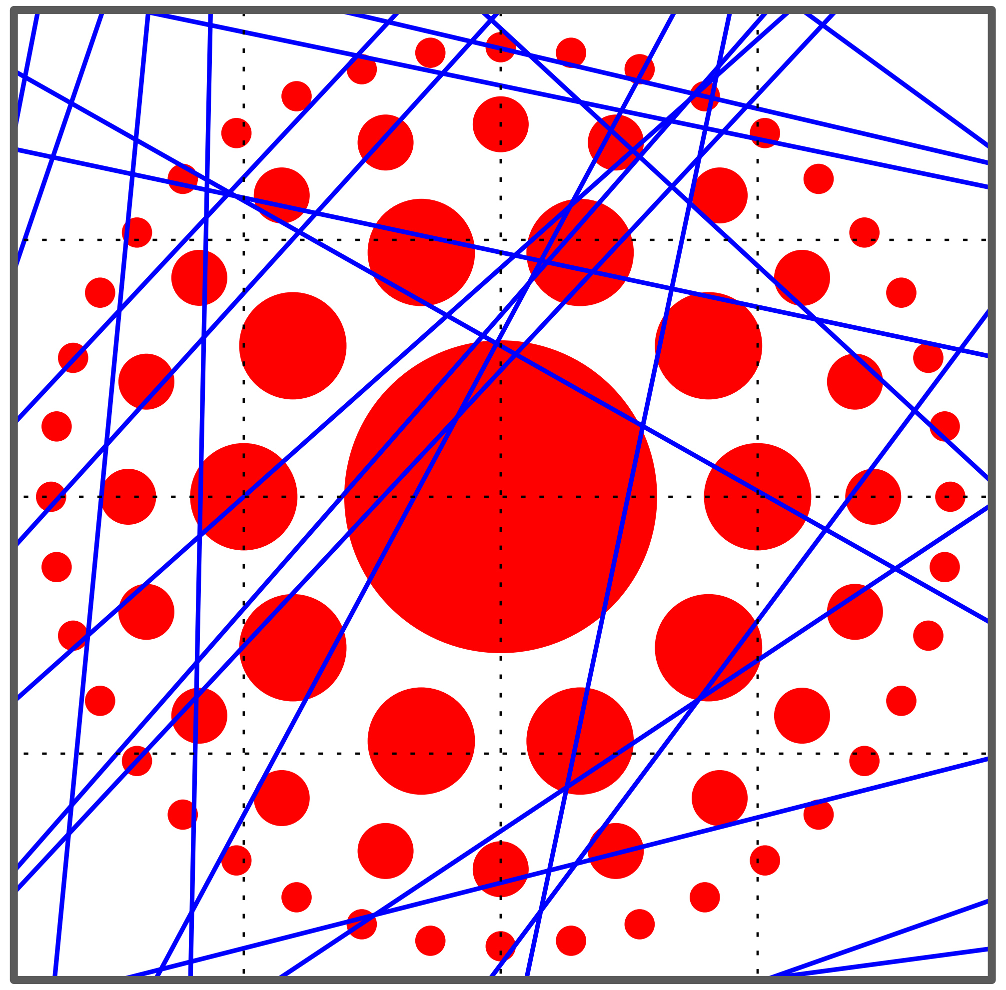
Modular Structure



Sierpinski Carpet
Variant 1



Sierpinski Carpet
Variant 2



Semi-annular
Structure

References

- [1] Herculano-Houzel, S. (2009). The Human Brain in Numbers: A Linearly Scaled-up Primate Brain. *Frontiers in Human Neuroscience, 3(31).*
- [2] Kleinberg, J. (2000). The Small-world Phenomenon: An Algorithmic Perspective. 163-170.
- [3] Liao, X., Vasilakos, A., & He, Y. (2017). Small-world human brain networks: Perspectives and challenges. Neuroscience & Biobehavioral Reviews. 77, 286-300.
- [4] Humpries, M. D. & Gurney, K. (2008). Network Small-Worldness: A quantitative Method for Determining Canonical Network Equivalanece. *PLoS ONE, 3*.
- [5] Pantone, R. D., Kendall J. D., & Nino, J. C. (2018). Memristive Nanowires Exhibit Small-World Connectivity. *Neural Networks*.
- [6] Lee, U., Oh,. G., Kim, S., Noh, G., Choi, B., & Mashour, G. A. (2011). Brain Networks Maintain a Scale-Free Organization across Consciousness, Anesthesia, and Recovery: Evidence for Adaptive Reconfiguration. *Anesthesiology, 113(5).*
- [7] Cohen, R. & Havlin, S. (2003). Scale-Free Networks Are Ultrasmall. *Physical Review Letters, 90(5).*