How can ideas from quantum computing improve or speed up neuromorphic models of computation?
Neuromorphic and Quantum Computing

• Two exciting new models for computation
  – Practical interest is driven by need for more efficient computational platforms
  – Specialized applications and processors are

• Neuromorphic computing
  – Adapt features of neural systems to computation

• Quantum computing
  – Adapt principles of quantum physics to computation

• Are there applications where the specialties of each model coincide?
  – Associate Memory Recall
Associative Memory

- **Associative Memory**
  - A data storage mechanism whereby locations are identified according to stored value

- **Content Addressable Memory (CAM)**
  - A memory that stores key-value pairs and recalls keys when provided with a value

- **Auto-associative CAM**
  - A CAM in which the key and value are the same

- **Random Access Memory (RAM)**
  - A memory that stores key-value pairs and recalls value when provided with a key/location

Pattern matching as a form of content addressable memory
Models of Associative Memory

• **Hopfield Networks**
  – An associative memory using a recurrent network of computational neurons
  – Network state evolves toward equilibrium

• **Discrete CAM model**
  – Consider a network of $n$ neurons, where the $i$-th neuron is in a bipolar state
    \[
    z_i \in \{ \pm 1 \}
    \]
  – Synaptic weight $w_{ij}$ couples neurons $i$ and $j$
    \[
    z_i = \begin{cases} 
    +1 & \text{if } \sum_j w_{ij} z_j > \theta_i \\
    -1 & \text{otherwise}
    \end{cases}
    \]
  – Each neuron is activated when its local field exceeds the activation threshold $\theta$
Content-Addressable Memory Recall

• Storing memories in a Hopfield network
  – The network’s synaptic weights store memories

  \[\xi^{(k)} \in \{\pm 1\}^n\]
  \[w_{ij} = \sum_{k=1}^{P} \xi_i^{(k)} \xi_j^{(k)}\]
  \[E(z; \theta) = -\frac{1}{2} \sum_{i,j=1}^{n} z_i w_{ij} z_j - \sum_{i=1}^{n} \theta_i z_i\]

• Recalling memories in a Hopfield network
  – Evolve under a stochastic update rule

  \[z_i' = \text{sign}\left(\sum_j w_{ij} z_j - \theta_i\right)\]
  – Memories are stable fixed-points
  – Convergence guaranteed because the network energy is a Lyapunov function
Mapping Recall to Optimization

• Recast update in terms of global optimization

\[ z'_i = \text{sign} \left( \sum_j w_{ij} z_j - \theta_i \right) \quad \leftrightarrow \quad z = \arg \min_z E(z; \theta) \]

- Search for the spin configuration that minimizes the network energy
- Thresholds (bias) still represent best guess
Adiabatic Quantum Optimization

• A quantum algorithm that returns the lowest energy state of a Hamiltonian
  — Evaluation makes use of the quantum superposition principle to sample configuration space
  — Execution depends on adiabatically evolving the quantum system toward a desired

\[ \hat{H}(t) = A(t)\hat{H}_0 + B(t)\hat{H}_1 \]

• Recovery of the lowest energy state is the primitive for memory recall

\[ E(z; \theta) \rightarrow \hat{H}_1 = -\sum_{i,j} w_{ij} \hat{Z}_i \hat{Z}_j - \sum_i \theta_i \hat{Z}_i \]
Experiments with Quantum Optimization

• **We use the D-Wave quantum processor**
  – A special purpose quantum processor that finds the ground state of an Ising Hamiltonian
  – Fabricated from coupled arrays of superconducting flux qubits
  – Operated as a quantum annealer
  – Validated as a probabilistic processor

• **Programmability limited by hardware constraints**
  – Size (# of qubits) and bits of precision restrict range of testable problem instances
  – Temperature and control systems limit range of execution scenarios
Experiments with Quantum Optimization

• Step 1: Specify problem instances
  – Select P memories and set one as the target memory
  – Calculate the synaptic weights using a learning rule L
  – Calculate the threshold / bias for the target memory

• Step 2: Solve problem instance
  – Program the Hopfield network into the hardware
  – Execute the quantum optimization program
  – Repeat execution N times to generate N samples

• Step 3: Confirm the correct memory was recalled.
  – Compute probability to recover correct memory
Experimental Measures of Capacity

Memory size 0.10n

- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias
Experimental Measures of Capacity

Memory size 0.30 n

- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias
Experimental Measures of Capacity

Memory size 0.50 n

- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias
Associative Memory Models with Adiabatic Quantum Optimization

• Concluding Points
  – Associative memory was modeled using a discrete Hopfield network
  – Memory recall in Hopfield networks was reduced to the quantum optimization problem
  – Recall was validated experimentally using the D-Wave processor
  – Recent theoretical arguments from Santra et al. suggest exponential capacity when using quantum optimizations (arxiv:1602.08149)
  – Our results find the accuracy is lower than expected, most likely due to hardware noise and constraints
  – Continued hardware advances will should address these limits
Associative Memory Models with Adiabatic Quantum Optimization

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