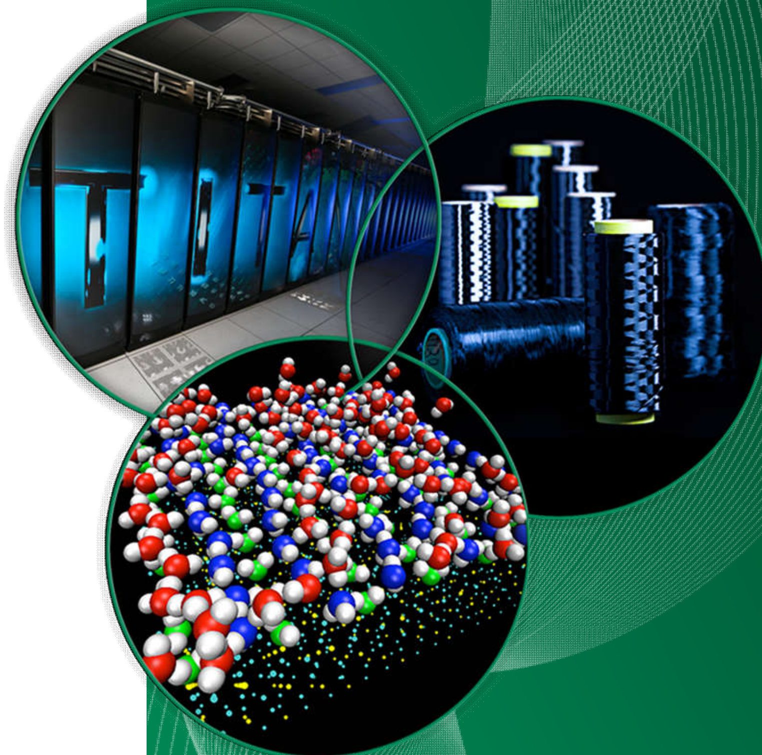
Neuromorphic Computation: Architectures, Models, Applications

Associative Memory Models with Adiabatic Quantum Optimization



Kathleen Hamilton, Alexander McCaskey, Jonathan Schrock, Neena Imam and Travis Humble

Quantum Computing Institute
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How can ideas from quantum computing improve or speed up neuromorphic models of computation?

Oak Ridge, Tennessee, 30 June 2016

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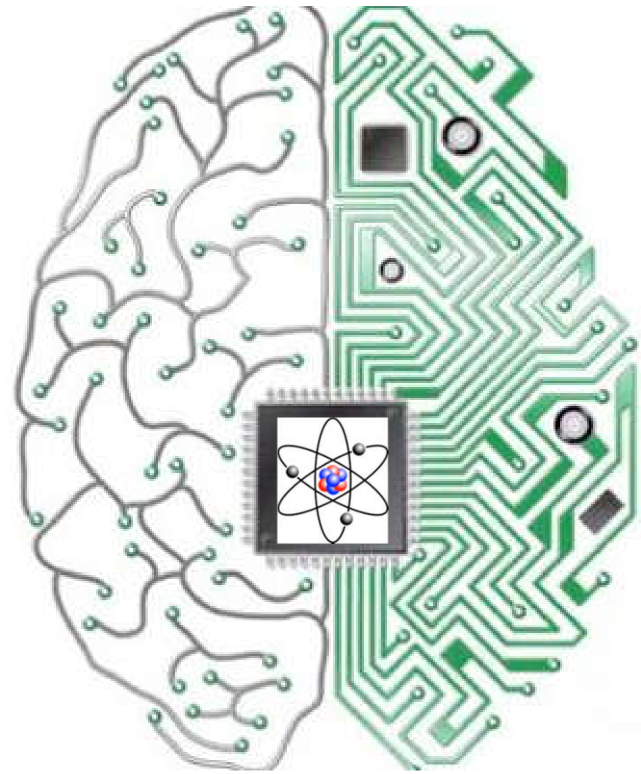
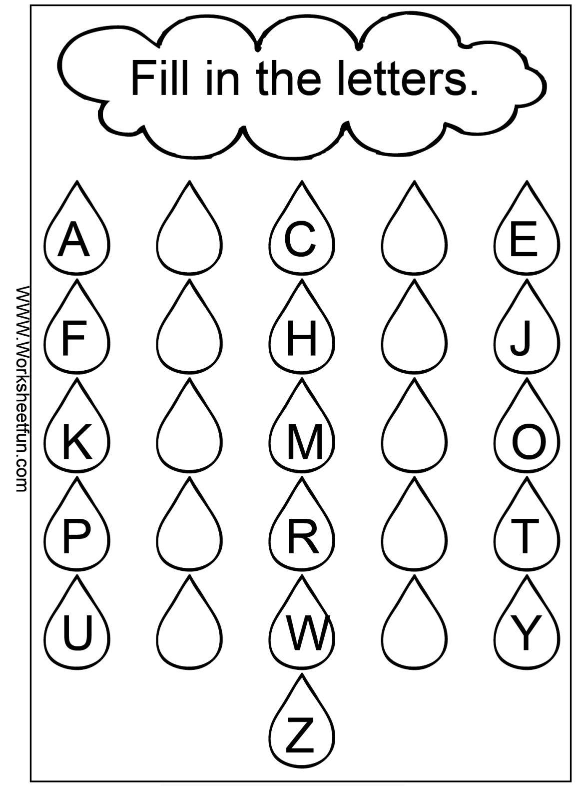
Neuromorphic and Quantum Computing

- **Two exciting new models for computation**
  - Practical interest is driven by need for more efficient computational platforms
  - Specialized applications and processors are
- **Neuromorphic computing**
  - Adapt features of neural systems to computation
- **Quantum computing**
  - Adapt principles of quantum physics to computation
- **Are there applications where the specialties of each model coincide?**
  - Associate Memory Recall



Associative Memory

- **Associative Memory**
  - A data storage mechanism whereby locations are identified according to stored value
- **Content Addressable Memory (CAM)**
  - A memory that stores key-value pairs and recalls keys when provided with a value
- **Auto-associative CAM**
  - A CAM in which the key and value are the same
- **Random Access Memory (RAM)**
  - A memory that stores key-value pairs and recalls value when provided with a key/location



Pattern matching as a form of content addressable memory

Models of Associative Memory

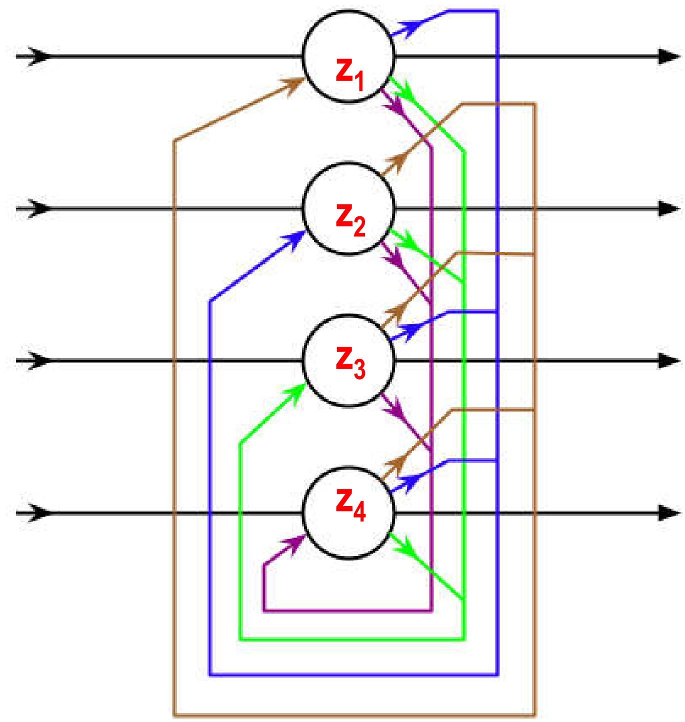
- **Hopfield Networks**
  - An associative memory using a recurrent network of computational neurons
  - Network state evolves toward equilibrium
- **Discrete CAM model**
  - Consider a network of *n* neurons, where the *i*-th neuron is in a bipolar state

$$z_i \in \{\pm 1\}$$

  - Synaptic weight w_{ij} couples neurons *i* and *j*

$$z_i = \begin{cases} +1 & \text{if} \sum_j w_{ij} z_j > \theta_i \\ -1 & \text{otherwise} \end{cases}$$

  - Each neuron is activated when its local field exceeds the activation threshold θ



A 4-neuron network showing connectivity between nodes

Content-Addressable Memory Recall

- **Storing memories in a Hopfield network**
  - The network's synaptic weights store memories

memories

$$\xi^{(k)} \in \{\pm 1\}^n$$

weights

$$w_{ij} = \sum_{k=1}^{P} \xi_i^{(k)} \xi_j^{(k)}$$

energy function

$$E(z;\theta) = -\frac{1}{2}\sum_{i,j=1}^{n} z_i w_{ij} z_j - \sum_{i=1}^{n} \theta_i z_i$$

- **Recalling memories in a Hopfield network**
  - Evolve under a stochastic update rule

$$z_i' = \text{sign}\left(\sum_j w_{ij} z_j - \theta_i\right)$$

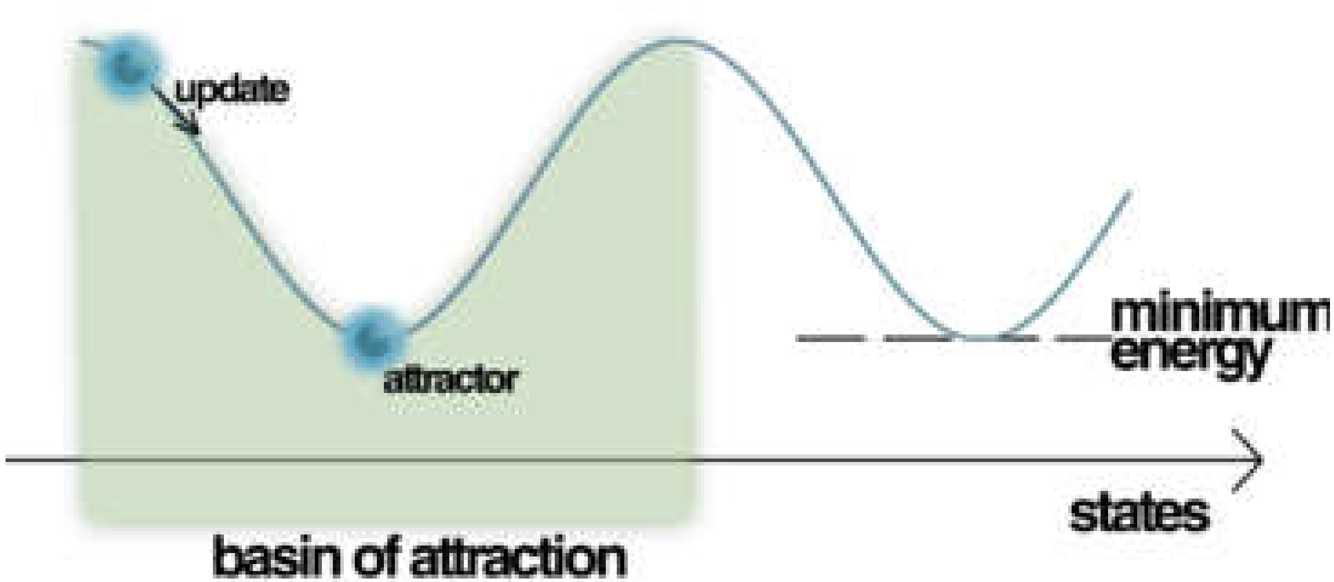
  - Memories are stable fixed-points
  - Convergence guaranteed because the network energy is a Lyapunov function

Mapping Recall to Optimization

- **Recast update in terms of global optimization**

$$z_i' = \mathrm{sign}\left(\sum_j w_{ij} z_j - \theta_i\right) \longleftrightarrow z = \arg\min_z E(z;\theta)$$

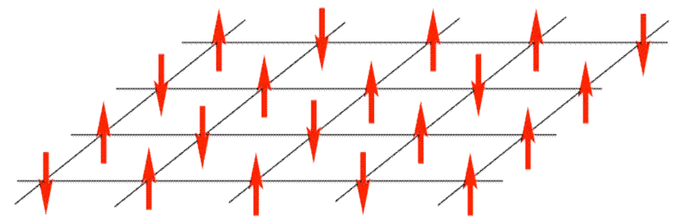
  – Search for the spin configuration that minimizes the network energy
  – Thresholds (bias) still represent best guess



Adiabatic Quantum Optimization

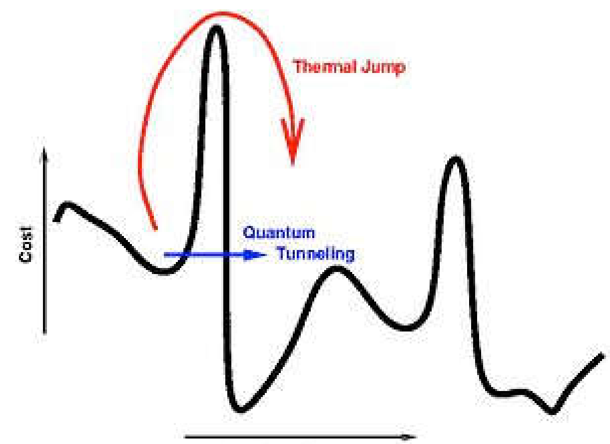
- **A quantum algorithm that returns the lowest energy state of a Hamiltonian**
  - Evaluation makes use of the quantum superposition principle to sample configuration space
  - Execution depends on adiabatically evolving the quantum system toward a desired

$$\hat{H}(t) = A(t)\hat{H}_0 + B(t)\hat{H}_1$$



- **Recovery of the lowest energy state is the primitive for memory recall**

$$E(z;\theta) \rightarrow \hat{H}_1 = -\sum_{i,j} w_{ij}\hat{Z}_i\hat{Z}_j - \sum_i \theta_i\hat{Z}_i$$



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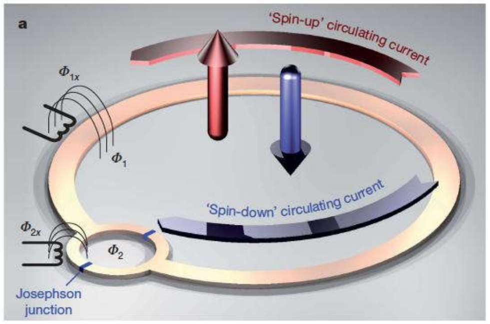
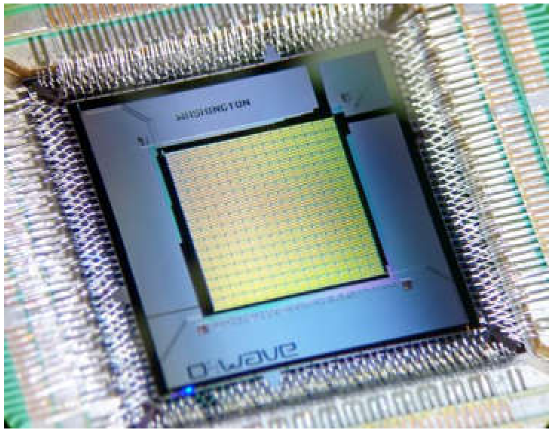
Experiments with Quantum Optimization

- **We use the D-Wave quantum processor**
  - A special purpose quantum processor that finds the ground state of an Ising Hamiltonian
  - Fabricated from coupled arrays of superconducting flux qubits
  - Operated as a quantum annealer
  - Validated as a probabilistic processor
- **Programmability limited by hardware constraints**
  - Size (# of qubits) and bits of precision restrict range of testable problem instances
  - Temperature and control systems limit range of execution scenarios





Experiments with Quantum Optimization

- **Step 1: Specify problem instances**
  - Select P memories and set one as the target memory
  - Calculate the synaptic weights using a learning rule L
  - Calculate the threshold / bias for the target memory
- **Step 2: Solve problem instance**
  - Program the Hopfield network into the hardware
  - Execute the quantum optimization program
  - Repeat execution *N* times to generate *N* samples
- **Step 3: Confirm the correct memory was recalled.**
  - Compute probability to recover correct memory

Problem

↓

Program

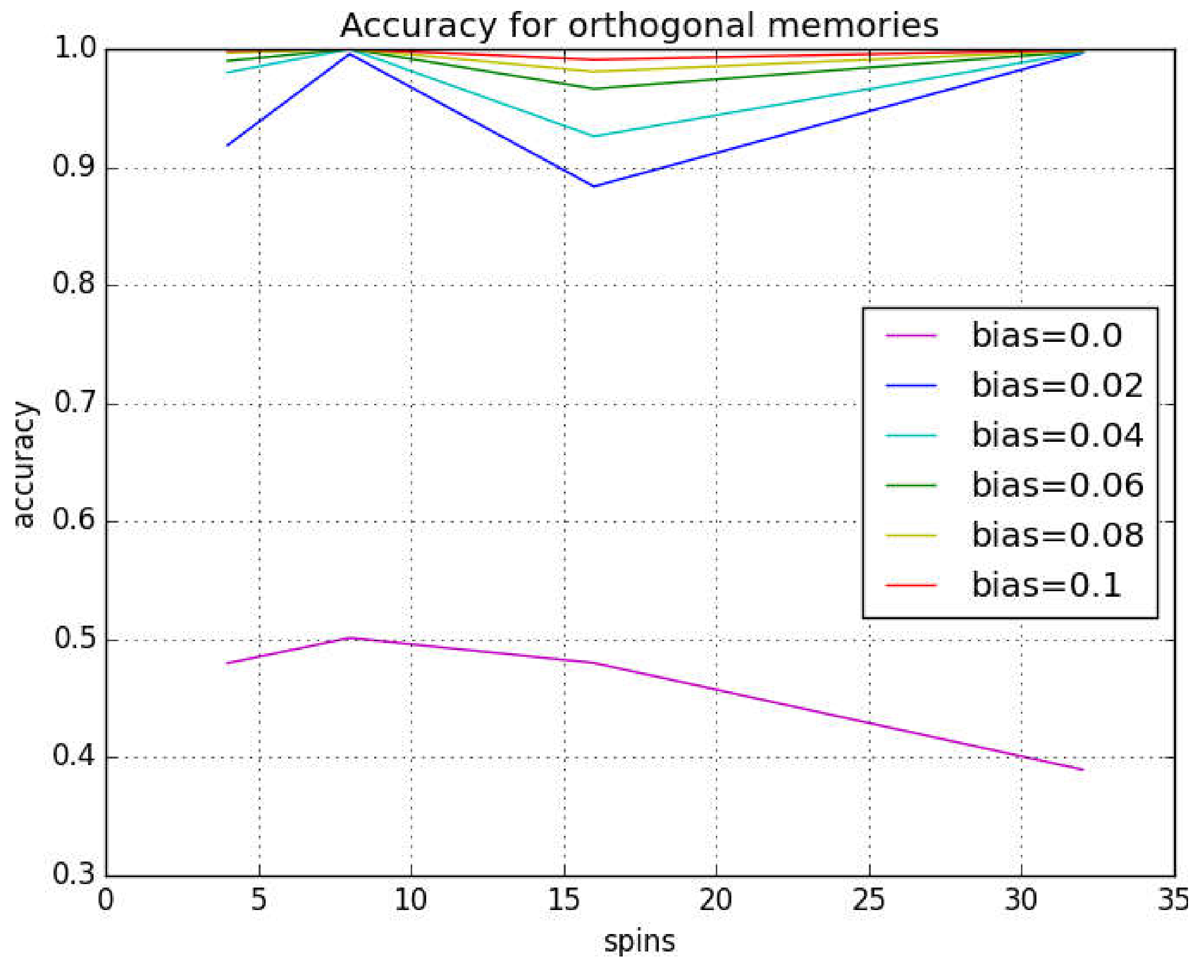
↓

Result

Experimental Measures of Capacity

Memory size 0.10n

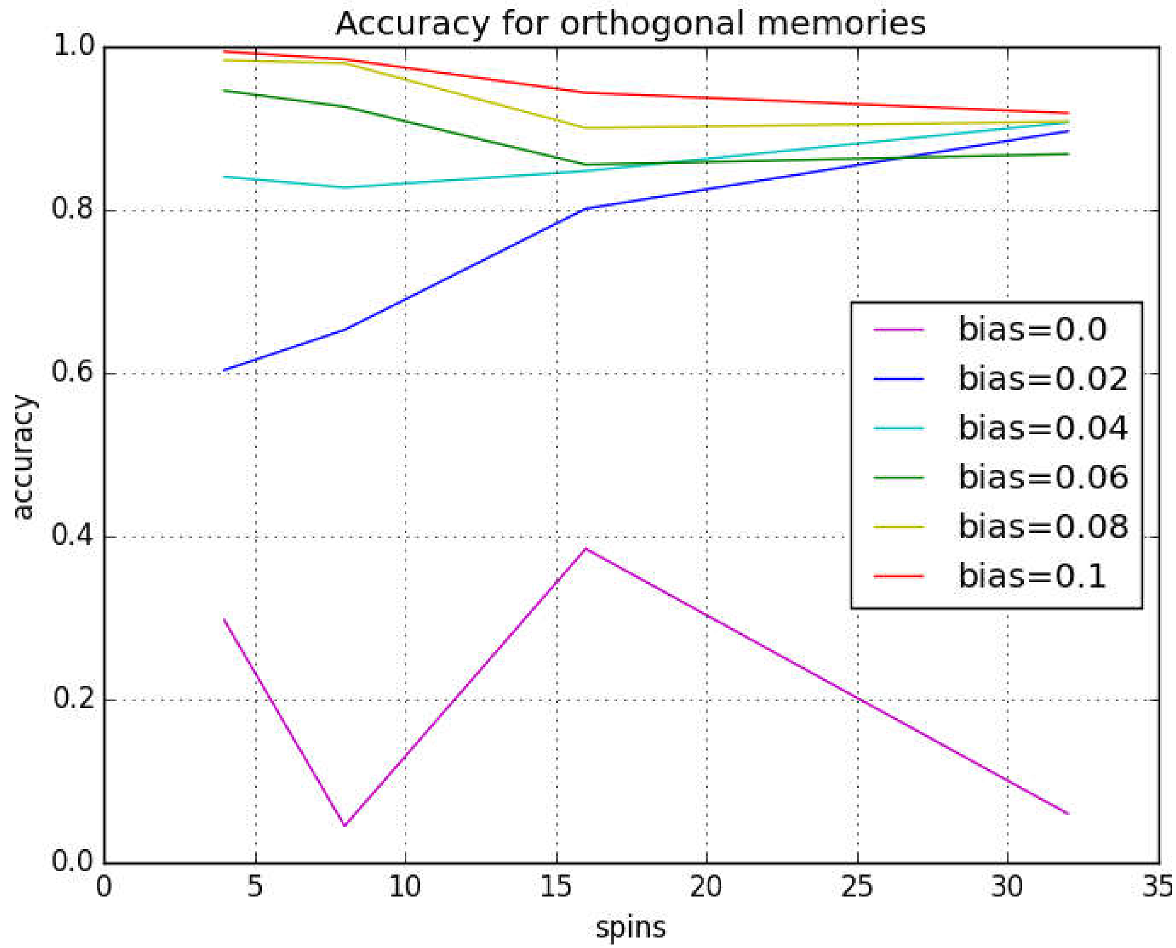
- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias



Experimental Measures of Capacity

Memory size 0.30 n

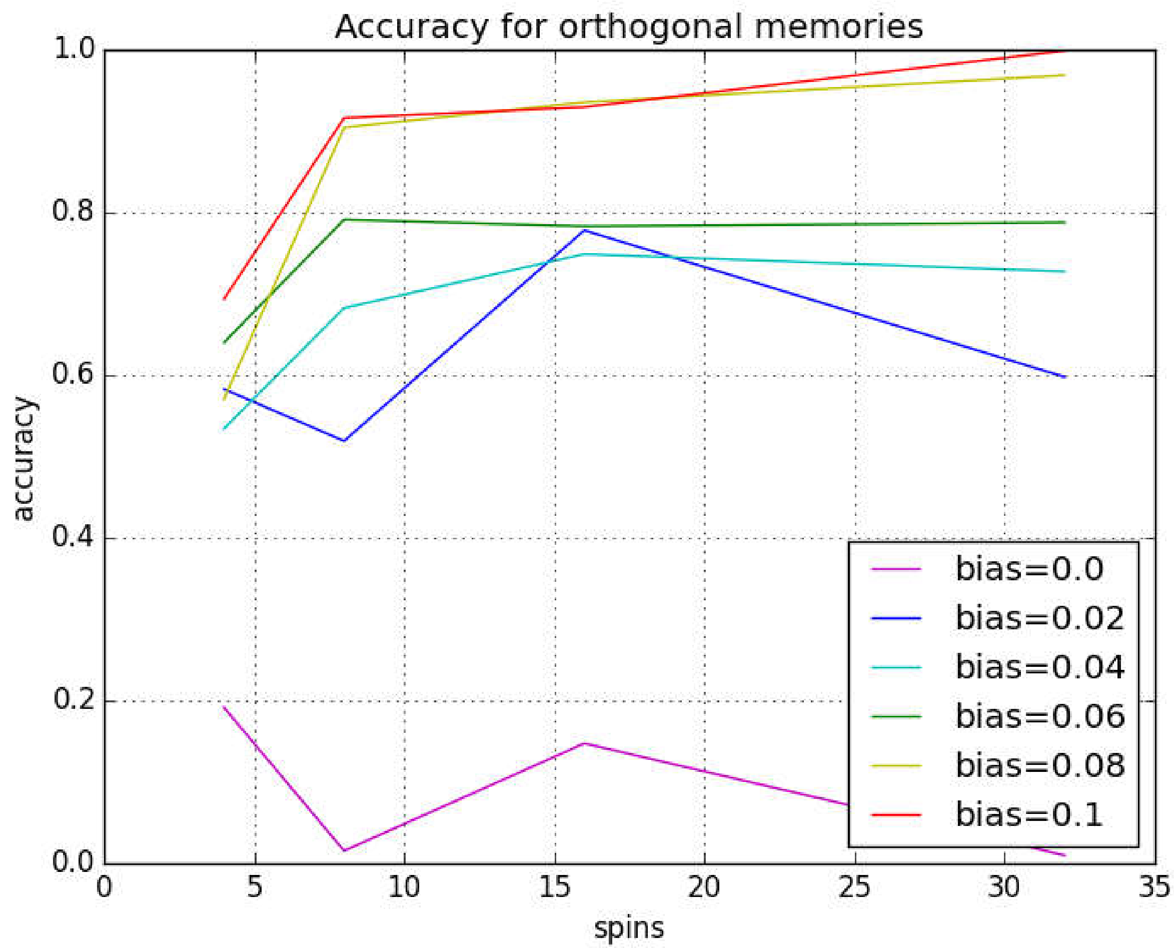
- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias



Experimental Measures of Capacity

Memory size 0.50 n

- Probability to recall memory correctly
- Average accuracy plotted
- 100 random instances per spin size
- Hebb learning rule
- Accuracy increases with increasing bias



Associative Memory Models with Adiabatic Quantum Optimization

- **Concluding Points**
  - Associative memory was modeled using a discrete Hopfield network
  - Memory recall in Hopfield networks was reduced to the quantum optimization problem
  - Recall was validated experimentally using the D-Wave processor
  - Recent theoretical arguments from Santra et al. suggest exponential capacity when using quantum optimizations (arixv:1602.08149)
  - Our results find the accuracy is lower than expected, most likely due to hardware noise and constraints
  - Continued hardware advances will should address these limits
  - Seddiqi and Humble, Adiabatic quantum optimization for associative memory recall, Frontiers in Physics 2, 79 (2014)

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